SOLUTIONS. Exam March 16, 2009
TSEI05 Analog and Discrete-time Integrated Circuits.

Exercise 1.

First notice that, because of symmetri, current through **M1**, **M3** and **M5** as well as current trough **M2**, **M4** and **M6** will be $I_D/2$.

- CMR=$[V_{in,min}, V_{in,max}]$.

$V_{in,min} = V_{GS6} + V_{GS4} + V_{SD1,min} - V_{SG1} = V_{eff6} + V_{tn} + V_{eff4} + V_{tn} + V_{eff1} - (V_{eff1} + V_{tp})$
$= \sqrt{\frac{I_D}{2\alpha_n}} + \sqrt{\frac{I_D}{2\alpha_n}} + 2V_{tn} - V_{tp} = 2\sqrt{\frac{I_D}{2\alpha_n}} + 2V_{tn} - V_{tp}$

$V_{in,max} = V_{DD} - V_{SD7,min} - V_{SG2} = V_{DD} - V_{eff7} - (V_{eff2} + V_{tp}) = V_{DD} - \sqrt{\frac{I_D}{\alpha_p}} - \sqrt{\frac{I_D}{2\alpha_p}} - V_{tp} =$
$= V_{DD} - (1+\sqrt{2})\sqrt{\frac{I_D}{2\alpha_p}} - V_{tp}$

- OR=$[V_{out,min}, V_{out,max}]$.
$V_{out,min} = V_{GS6} + V_{GS4} + V_{SD1,min} - V_{SD2,min} = V_{eff6} + V_{tn} + V_{eff4} + V_{tn} + V_{eff1} - V_{eff2}$
$= \sqrt{\frac{I_D}{2\alpha_n}} + V_{tn} + \sqrt{\frac{I_D}{2\alpha_n}} + V_{tn} + \sqrt{\frac{I_D}{2\alpha_p}} - \sqrt{\frac{I_D}{2\alpha_p}} = 2\sqrt{\frac{I_D}{2\alpha_n}} + 2V_{tn}$

$V_{out,max} = V_{DD} - V_{SD7,min} - V_{SD1,min} - V_{GS4} + V_{DS4,min} = V_{DD} - V_{eff7} - V_{eff1} - (V_{eff4} + V_{tn}) + V_{eff4} = V_{DD} - \sqrt{\frac{I_D}{\alpha_p}} - \sqrt{\frac{I_D}{2\alpha_p}} - V_{tn} = V_{DD} - (1+\sqrt{2})\sqrt{\frac{I_D}{2\alpha_p}} - V_{tn}$

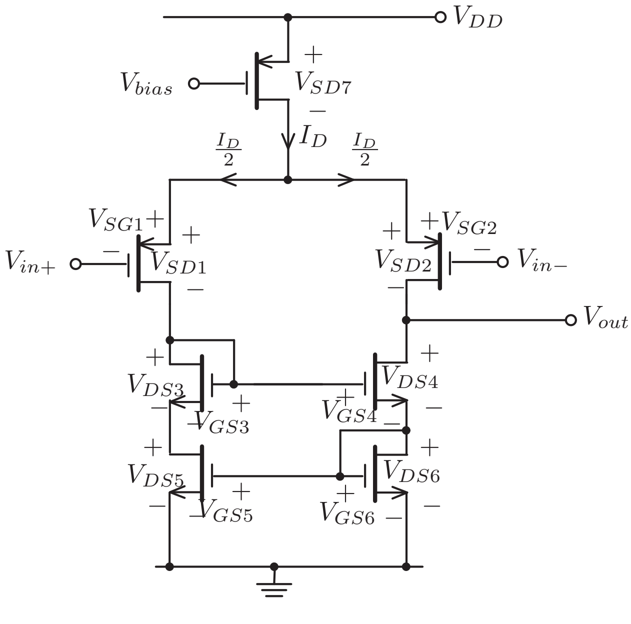

Figure 1: Amplifier.

Answer:
CMR=$\left[2\sqrt{\frac{I_D}{2\alpha_n}} + 2V_{tn} - V_{tp}, V_{DD} - (1+\sqrt{2})\sqrt{\frac{I_D}{2\alpha_p}} - V_{tp}\right]$

OR=$\left[2\sqrt{\frac{I_D}{2\alpha_n}} + 2V_{tn}, V_{DD} - (1+\sqrt{2})\sqrt{\frac{I_D}{2\alpha_p}} - V_{tn}\right]$

Exercise 2.

a) **Figure 2** shows a small signal equivalent. As the bias current sources (I_{01} and I_{02}) are ideal they will be replaced by open circuits in the small signal equivalent circuit.

   Notice that the current through g_{ds2} must be $g_{m2}V_{gs2}$. Then the current through g_{ds1} must be $g_{m1}V_{in}$. (Also the current through g_{ds3} must be $g_{m3}V_{gs3}$.)

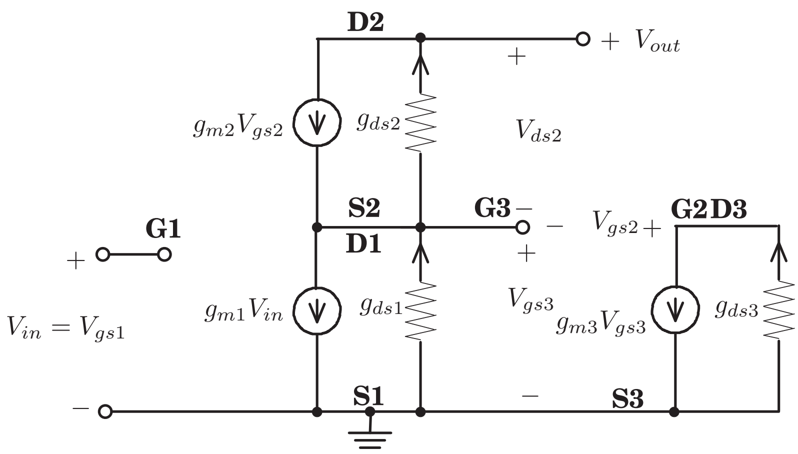

Figure 2: Small signal equivalent.

From the small signal equivalent circuit we get:

$$V_{out} = V_{ds2} + V_{gs3} = -\frac{1}{g_{ds2}} g_{m2} V_{gs2} - \frac{1}{g_{ds1}} g_{m1} V_{in} \quad (1)$$

$$V_{gs2} = V_{G2} - V_{S2} = V_{G2} - V_{G3} = -\frac{1}{g_{ds3}} g_{m3} V_{gs3} - V_{gs3} = -V_{gs3}(1 + \frac{g_{m3}}{g_{ds3}}) \quad (2)$$

$$V_{gs3} = -\frac{1}{g_{ds1}} g_{m1} V_{in} \quad (3)$$

Equations (1)-(3) gives:

$$V_{out} = \left(\frac{g_{m2}}{g_{ds2}}(1 + \frac{g_{m3}}{g_{ds3}}) + 1\right)\left(-\frac{g_{m1}}{g_{ds1}} V_{in}\right) \quad (4)$$

Assuming that $g_m >> g_{ds}$ equation (4) gives the transfer function:

$$\underline{\underline{H(s) = \frac{V_{out}}{V_{in}} = -\frac{g_{m1}}{g_{ds1}} \cdot \frac{g_{m2}}{g_{ds2}} \cdot \frac{g_{m3}}{g_{ds3}}}} \quad (5)$$

b) As $g_m \propto \sqrt{\frac{W}{L} I_D}$ and $g_{ds} \propto \frac{I_D}{L}$ we get from equation (5):

$$H = K \cdot \frac{\sqrt{\frac{W_1}{L_1} I_{01}} \cdot \sqrt{\frac{W_2}{L_2} I_{01}} \cdot \sqrt{\frac{W_3}{L_3} I_{02}}}{\frac{I_{01}}{L1} \cdot \frac{I_{01}}{L2} \cdot \frac{I_{02}}{L3}} \quad (6)$$

As all $L_i = L$ equation (6) can be written:

$$H = K \cdot \frac{\frac{\sqrt{W_1 W_2}}{L} \cdot I_{01} \cdot \sqrt{\frac{W_3}{L} I_{02}}}{\frac{I_{01}^2}{L^2} \cdot \frac{I_{02}}{L}} = \frac{\sqrt{W_1 W_2 W_3 L^3}}{I_{01}\sqrt{I_{02}}} \quad (7)$$

Answer: $\underline{\underline{f = \frac{\sqrt{W_1 W_2 W_3 L^3}}{I_{01}\sqrt{I_{02}}}}}$

Exercise 3.

A: **M1** forms current mirrors together with **M5**, **M7** and **M9**, giving the gainstages **B**, **C** and **D** appropriate current. The current through **M10** can be changed by changing $R_{offchip}$.

B: Forms a differential gain-stage. Gives the OP-amp. high input resistans. Gives also high gain.

C: **M6** and **M7** forms a common source stage (CS-stage), giving high voltage gain.

Capacitor C_c is a feed-back capacitor that increases phase-margin. (Compensation capacitor.)

D: **M8** and **M9** forms a common drain stage (CD-stage) giving low output resistans to the OP-amp. (The CS-stage has a high ouput resistans, which means that excluding stage D we have an OTA, but in an OP-amp. we want low output resistans.)

Exercise 4.

To determine the spectral density R_{out} on the output, caused by flicker noise at the gate of transistor **M2**, you first have to determine the transfer function from **G2** to output. I.e. you have to determine $H(\omega) = V_{out}/V_{sg2}$ when V_{in} is short-circuited. Then you introduce the noise source between **G2** and **S2**.
If the flicker noise has spectral density R_n then $R_{out} = \mid H(\omega) \mid^2 R_n$.

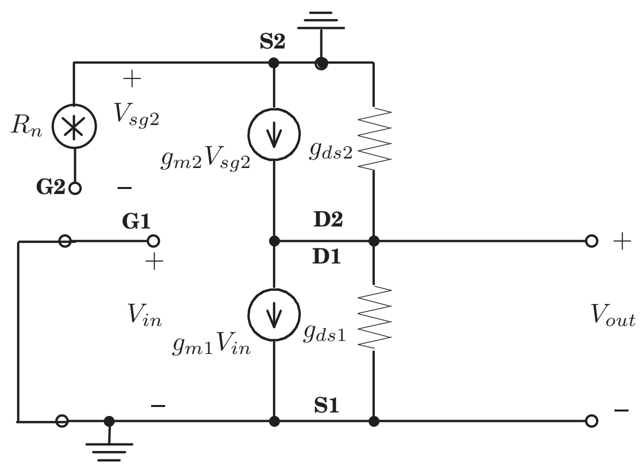

Figure 3: Small signal equivalent circuit.

As $V_{in} = 0$, $g_{m1}V_{in} = 0$. V_{out} then can be written

$$V_{out} = g_{m2}V_{sg2} \cdot \frac{1}{g_{ds1} + g_{ds2}} \Rightarrow H = \frac{V_{out}}{V_{sg2}} = \frac{g_{m2}}{g_{ds1} + g_{ds2}}$$

From the formulas at page 7 we have: $R_n = \dfrac{K_2}{W_2 L_2 C_{ox2} f}$

Thus

$$\textbf{Answer: } R_{out}(f) = \frac{g_{m2}^2}{(g_{ds1} + g_{ds2})^2} \cdot \frac{K_2}{W_2 L_2 C_{ox2} f}$$

Exercise 5

In Figure 4 a) below current $I_x = I_f$ and voltage $V_- = 0$ because OP-amp. is ideal. The voltage over R_f then will be $V_{out} - 0$.

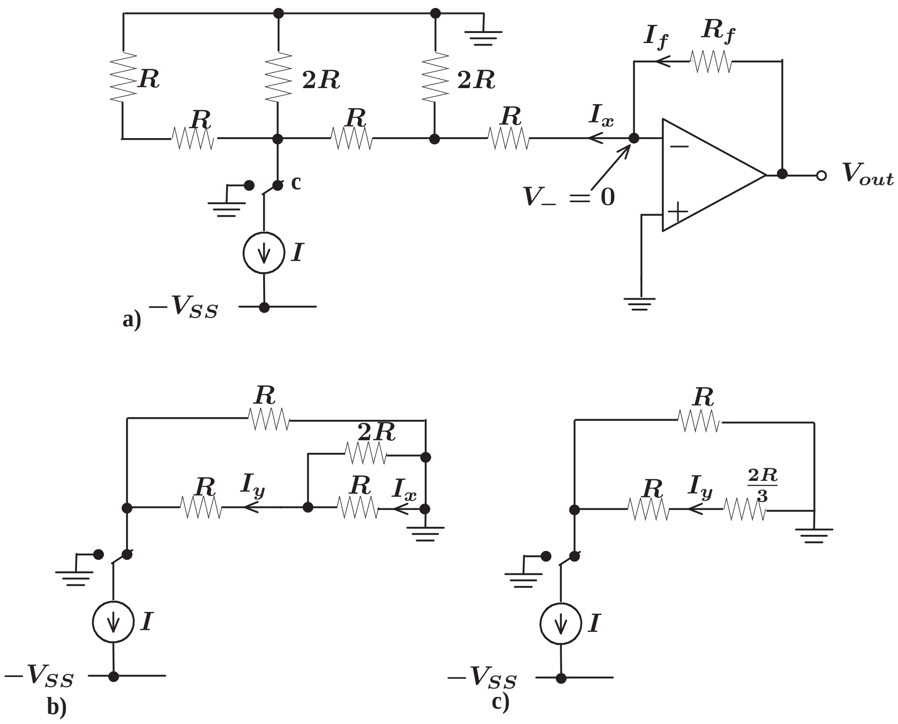

Figure 4: D/A-converter.

$$I_x = I_f = \frac{V_{out} - 0}{R_f} \Rightarrow V_{out} = R_f I_f$$

As $V_- = 0$ (virtual ground) the R, $2R$ resistor net can be redrawn as figures 4 b) and 4 c) show, to get the relation between currents I_x, I_y and I.
Current division in Figure 4 c) gives:

$$I_y = \frac{R}{R + R + \frac{2}{3}R} \cdot I = \frac{3}{8} I$$

Using current division and noticing that $I_x = I_f$ Figure 4 b) gives:

$$I_f = I_x = \frac{2R}{R + 2R} \cdot I_y = \frac{2}{3} I_y \Rightarrow I_f = \frac{2}{3} \cdot \frac{3}{8} I = \frac{1}{4} I$$

As $V_{out} = R_f I_f$ we have, finally

$$V_{out} = R_f \frac{1}{4} I$$

Answer: $\underline{\underline{V_{out} = R_f \frac{I}{4}}}$