# Written Test <br> TSTE80, Analog and Discrete-time Integrated Circuits 

| Date | May 28, 2003 |
| :--- | :--- |
| Time: | $8-12$ |
| Max. no of points: | $70 ;$ <br> 40 from written test, <br> 15 for project, and 15 for assignments. |
| Grades: | 30 for 3, 42 for 4, and 56 for 5. |
| Allowed material: | All types of calculators except Lap Tops. All types of <br> tables and handbooks. The textbook Johns \& Martin: |
|  | Analog Integrated Circuit Design. |
| Examiner: | Lars Wanhammar. |
| Responsible teacher: | Robert Hägglund. <br> Tel.: 0705-48 56 88. |
| Correct (?) solutions: | Solutions and results will be displayed in House B, <br> entrance $25-27$, ground floor. |

## Good Luck!

## Student's Instructions

The CMOS transistor operation regions, small signal parameters, and noise characteristics are found on the last page of this test.

Generally, do not just answer yes or no to a short question. You always have to answer with figures, formulas, etc., otherwise no or fewer points will be given.

Basically, there are few numerical answers to be given in this test.
You may write down your answers in Swedish or English.

## Exercise

## 1. Large-signal analysis

The circuit in Figure 1.1 is a commonly used structure when designing analog circuits. In all following exercises assume that transistor M1 is biased in saturation. Also assume that the $W / L$ ratio of transistor M 2 is $X$ times larger than that of transistor M1, i.e., $W_{2} / L_{2}=X W_{1} / L_{1}$.
a) Derive the output voltage as a function of the factor $X$, i.e., $V_{\text {out }}=f(X)$, when transistor M2 is saturated. Express the output voltages in terms of the current $I_{0}$ and transistor design parameters, but not voltages other than the power supply voltage.
b) Derive the output voltage as a function of the factor $X$, i.e., $V_{o u t}=g(X)$, when transistor M2 is operating in the linear region. Express the output voltages in terms of the current $I_{0}$ and transistor design parameters, but not voltages other than the power supply voltage.
c) Determine for which value of $X$ transistor M2 switches from operating in the saturation region to the linear region.
(2p)
d) Sketch the output voltage as a function of the $X$, i.e., $V_{o u t}=h(X)$, for $X>0$.
(1p)


Figure 1.1 A simple current mirror.

## 2. Small-signal analysis

The circuit in Figure 2.1 is going to be implemented in a CMOS process. The parasitic of interest is the gate-source capacitor. The feedback amplifier has a gain of $-A$ where $A$ is positive and $A » 1$. Further, assume that $C_{G S, 2}<C_{L}$.
a) Derive the transfer function, i.e., $V_{\text {out }} / V_{\text {in }}$, of the circuit shown in Figure 2.1. Do not neglect the bulk effects.
b) Derive expressions for the DC gain, first pole, second pole, possible zeros, and the unity-gain frequency in terms of $I_{1}, I_{2}, W_{1}, W_{2}, L_{1}$, and $L_{2}$. Neglect the influence of the bulk effect.
(4p)
c) How is the phase margin effected if the bias current $I_{1}$ is increased, i.e., how is the phase effected at the unity-gain frequency of the circuit?(2p)


Figure 2.1 A CMOS circuit.

## 3. Noise analysis

An operational amplifier is used in a CMOS circuit. Assume that the OPamp is ideal.
a) Derive the transfer function of the circuit shown in Figure 3.1.
b) Derive the equivalent thermal output noise spectral density of the circuit. Assume that the operational amplifier is noiseless while the resistors generate thermal noise according to $V_{R}^{2}=4 k T R$.


Figure 3.1 An operational amplifier configuration.

## 4. Switched-capacitor circuit analysis

A switched capacitor circuit in clock phase 1 is shown in Figure 4.1. The input signal is sampled according to $V_{i n}(t)=V_{i n}(t+\tau)$. Express the output voltage, $V_{\text {out }}(z)$, for clock phase 1 of the switched capacitor circuit shown in Figure 4.1. Assume that the OTA is ideal.
(8p)


Figure 4.1 A switched-capacitor circuit.
5. A mixture of questions
a) You have designed and fabricated a digital-to-analog converter with 62 dB SNR, where the noise power is dominated by the quantization noise. The application requires 86 dB SNR. How can you increase the SNR of the data converter?
b) Derive expressions for the common-mode and the output ranges of the circuit shown in Figure 5.1 in terms of currents, $I_{D 5}$ and $I_{D 3}$, and transistor parameters. The amplifier circuit is symmetric with respect to the transistor sizes, i.e., $W_{1} / L_{1}=W_{2} / L_{2}$ and so on.


Figure 5.1 A CMOS amplifier structure.
c) A three terminal switch, Figure 5.2(a), is realized with two PMOS devices, Figure 5.2(b), in an SC circuit. The gates of the transistors are connected to the clocks $\phi_{1}$ and $\phi_{2}$, respectively. The waveforms for two different types of 2-phase clocks are shown in Figure 5.2(c) and (d), where $\phi_{1}$ is solid and $\phi_{2}$ is dashed. Which of these two 2-phase clocks ((c) or (d)) should be used in order to guarantee a good operation of the SC circuit. Motivate your answer carefully.
x

(a)

(b)

(c)

(d)

Figure 5.2 (a) A schematic view of a switch within an SC circuit. (b) Transistor implementation of the switch. (c) and (d) two different 2-phase clocks.

## Transistor formulas and noise

## CMOS transistors

## Current formulas and operating regions

Cut-off:

$$
V_{G S}<V_{T}
$$

$$
I_{D} \approx 0
$$

Linear:

$$
V_{G S}-V_{T}>V_{D S}>0 \quad I_{D} \approx \frac{\mu_{0} C_{o x}}{2} \cdot \frac{W}{L} \cdot\left(2\left(V_{G S}-V_{T}\right)-V_{D S}\right) \cdot V_{D S}
$$

Saturation:

$$
0<V_{G S}-V_{T}<V_{D S} \quad I_{D} \approx \frac{\mu_{0} C_{o x}}{2} \cdot \frac{W}{L} \cdot\left(V_{G S}-V_{T}\right)^{2} \cdot\left(1+\lambda V_{D S}\right)
$$

## Small-signal parameters

Linear region:

$$
g_{m} \approx \mu_{0} C_{o x} \cdot \frac{W}{L} \cdot V_{D S} \quad g_{d s} \approx \mu_{0} C_{o x} \cdot \frac{W}{L} \cdot\left(V_{G S}-V_{T}-V_{D S}\right)
$$

Saturation region:

$$
g_{m}=\frac{d I_{D}}{d V_{G S}} \approx \sqrt{2 \mu_{0} C_{o x} \frac{W}{L} I_{D}} \quad g_{d s}=\frac{d I_{D}}{d V_{D S}} \approx \lambda I_{D}
$$

## Circuit noise

## Thermal noise

The thermal noise spectral density at the gate of a CMOS transistor is

$$
\frac{\overline{v^{2}}}{\Delta f}=\frac{8 k T}{3} \cdot \frac{1}{g_{m}}
$$

## Flicker noise

The flicker noise spectral density at the gate of a CMOS transistor is

$$
\frac{\overline{v^{2}}}{\Delta f}=\frac{K}{W L C_{o x} f}
$$

