Lesson 8

Lesson Exercises: **K27, K28, K36, K37, K38**

Recommended Exercises: *K24, K25, K26, K30, K39, B10.1-5*

Theoretical Issues: **SC-filter, Laddningsanalys**

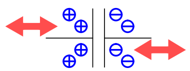
Theoretical

• Switched-Capacitor Circuit Technique, SC

The advantages of not having to implement on-chip resistances are several. In the previous lesson we saw that the resistance implemented with a transistor is signal dependent. There are certain processes allowing special poly layers to implement resistors. There are however problems with matching and parasitic capacitances. The SC technique utilizes the fact that capacitor ratios are used. Then we only need to match capacitors.

To know all the principles of the SC technique, we have to consider the charge redistributiuon that occurs in the circuits.

Charge redistribution analysis

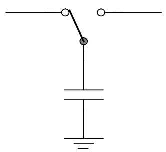


Consider a capacitor. The charge is equalt to the voltage over the plates times the capacitance value (constant):

$$Q = CV$$

By noting the amount of charge that is transferred between different capacitor plates, a flow chart for the charge (and thereby voltages) can be constructed. By only allowing the charge to move at certain time intervals, at discrete-time points, we can control the behaviour of the circuit.

Equivalent Resistance

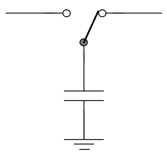


Consider the capacitance and the switch at time t. The charge on the top plate is equal to

$$q(t) = C \cdot v_1(t)$$

A certain amount of charge will flow from the input to the top plate.

$$\Delta q(t) = q(t) - q(t - \tau) = C[v_1(t) - v_2(t - \tau)]$$



At time $t + \tau$ the charge is given by

$$q(t + \tau) = C \cdot v_2(t + \tau)$$

The charge floating from the output to the top plate is given by

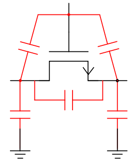
$$\Delta q(t + \tau) = C[v_2(t + \tau) - v_1(t)]$$

From this we conclude that during a clock period, T, a certain charge, Δq, will flow from v_1

to v_2. This charge must equal $\Delta q = C \cdot (V_1 - V_2)$ the capacitance and change of voltage between the terminals. If there is no difference, no charge will be transferred, etc. The average current, $I = q \cdot T$, gives

$$V_1 - V_2 = \frac{T}{C} \cdot I \text{ which gives the equivalent resistance } R \equiv \frac{T}{C}$$

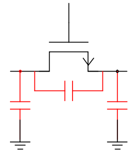
Parasitic capacitances



We can associate a parasitic capacitance with all terminals of the transistor, source, drain, gate, and bulk:

$$C_{gd}, C_{gs}, C_{ds}, C_{db} \text{ and } C_{sb}$$

The switching signal is considered to be ac grounded and C_{gd} is coupled in parallel with C_{db}, as well as C_{gs} with C_{sb}.



In most cases the influence of C_{ds} is neglected, due to its low value. When the switch is conducting, C_{ds} is also considered to be replaced with a short.

By noting these parasitics their influence on the total transfer function can be analyzed.

Discrete-time Spectrum

The discrete-time signal can be written as

$$y(t) = \sum_{k=0}^{\infty} y(kT)[u(t - kT) - u(t - (k+1)T)]$$

where T is the clock period. The output spectrum can be written as

$$Y(\omega) = \mathrm{sinc}(\omega T) \cdot Y[\omega T]$$

Tips for charge redistribution

Charge can not disappear fron an unconnected plate

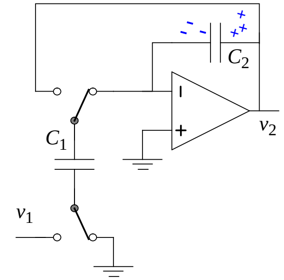
On a voltage controlled operational amplifier it is only the output that can add or remove charge. The input is coupled to transistor gates, wherein no current can flow.

The charge disappears from the capacticance if both plates are connected to the same potential (short cut).

The charge redistribution is done in discrete events

If a capacitance is switched to a charged capacitance net, the charge will move and eventually reach equilibrium. By using the tips above and use the knowledge of how the charge is stored from one event to another, the transfer function can be derived.

Example Charge I (K25)



Derive the transfer function and discuss the sensitivity of the circuit. Values are

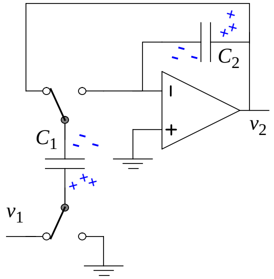
$$C_1 = C_2 \text{ and } C_1 = 1.12C_2$$

Consider the start-up conditions at time t. The charge at C_1 and C_2 is

$$q_1(t) = 0 \text{ and } q_2(t) = C_2 v_2(t)$$

C_1 is coupled between ground and virtual ground (OPamp input). The charge must be zero.

Time $t + \tau$. Switches have changed.



C_1 is charged by the voltage $v_1(t + \tau)$ and the output of the OPamp, v_2, that adds extra charge. The charge at C_1 becomes

$$q_1(t + \tau) = C_1[v_1(t + \tau) - v_2(t + \tau)]$$

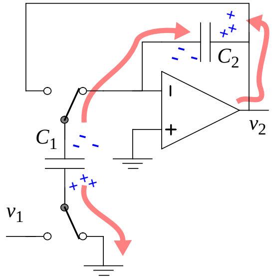
Note the chosen sign of the charge. For C_2 we have

$$q_2(t + \tau) = C_2 v_2(t + \tau).$$

On the negative plate, the charge is stored.

$$q_2(t + \tau) = q_2(t) \text{ dvs } v_2(t + \tau) = v_2(t)$$

(No charge can disappear from the input of the OPamp if it is unconnected).



At time $t + 2\tau$ the switches are closed. C_1 is again connected to ground and virtual ground, which empties C_1. The positive charge leaks down to ground, the negative charge is redistributed to the negative plate of C_2. The extra charge needed to compensate the positive plate of C_2 is taken from the OPamp output.

The charge at C_1 and C_2 must be

$$q_1(t + 2\tau) = 0 \text{ and } q_2(t + 2\tau) = C_2 v_2(t + 2\tau)$$

Charge conservation gives (at the negative plate of C_2)

$$-q_2(t + 2\tau) = -q_2(t + \tau) + (-q_1(t + \tau)) = -q_2(t) - q_1(t + \tau)$$

This gives

$$C_2 v_2(t + 2\tau) = C_2 v_2(t) + C_1[v_1(t + \tau) - v_2(t + \tau)] =$$
$$= C_2 v_2(t + \tau) + C_1[v_1(t + \tau) - v_2(t + \tau)]$$

We also see that

$$v_2(t + 2\tau) = v_2(t + 3\tau)$$

which gives

$$C_2 v_2(t + 3\tau) - C_2 v_2(t + \tau) + C_1 v_2(t + \tau) = C_1 v_1(t + \tau)$$

z-transform, with $t = kT$ and $2\tau = T$

$$[C_2 z^{3/2} - C_2 z^{1/2} + C_1 z^{1/2}]V_2(z) = C_1 z^{1/2} V_1(z)$$

which gives the transfer function

$$H(z) = \frac{V_2(z)}{V_1(z)} = \frac{C_1}{C_2} \cdot \frac{z^{1/2}}{z^{3/2} - (1 - C_1/C_2)} = \frac{C_1}{C_2} \cdot \frac{1}{z - (1 - C_1/C_2)}$$

If the capacitances are equally large, $C_1 = C_2$, the circuit is a simple delay element, (sample-and-hold)

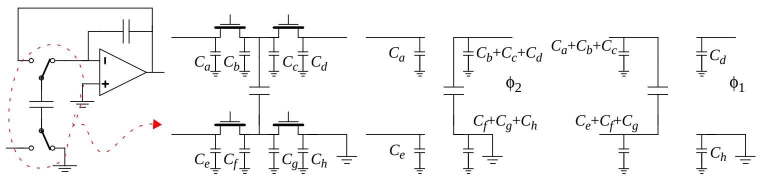
$$H(z) = z^{-1}$$

In the second case, $C_1 = 1.12C_2$, the transfer function becomes

$$H(z) = \frac{1.12}{z + 0.12}$$

This is used to compensate for the sinc weighting of the signal.

Example parasitics I



The parasitic capacitances are associated with all nodes in the circuit. Consider the parasitic capacitances, C_a through C_h. They are the parasitic capacitances associated with the switches as discussed earlier.

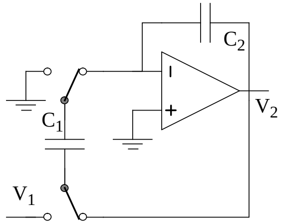
During clock phase ϕ_2, C_b, C_c and C_d are coupled in parallel. The same is true for C_f, C_g and C_h. C_a is connected to the output of the OPamp. C_e is connected to the input signal. The previous charge at the capacitances coupled in parallel will redistribute to C_2.

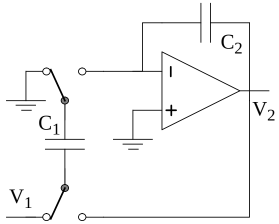
During clock phase ϕ_1, C_a, C_b and C_c are coupled in parallel. The same is true for C_e, C_f and C_g. C_d is coupled to virtual ground at the OPamp input. C_h is connected to ground. The parallel capacitances will be charged and during next clock phase this charge redistribute and affect the transfer function.

Note that C_h and C_d always are connected to ground or virtual ground and will therefore not affect the transfer function. While the input signal is directly connected to C_1 the capacitances

C_e, C_f, C_g, C_h will not affect the transfer function.

Example Charge II (K26)





Consider time t. Charge at C_1 and C_2 is

$$q_1(t) = C_1 v_2(t) \text{ and } q_2(t) = C_2 v_2(t)$$

At $t + \tau$ C_1 is charged with $v_1(t)$

$$q_1(t+\tau) = C_1 v_1(t+\tau)$$

C_2 conserves its charge

$$q_2(t) = C_2 v_2(t) = q_2(t+\tau) = C_2 v_2(t+\tau)$$

At time $t + 2\tau$ C_1 is switched

$$q_1(t+2\tau) = C_1 v_2(t+2\tau)$$

The charge at C_1 is redistributed between C_2 and C_1 in such a way that the total charge is conserved

$$q_1(t+2\tau) + q_2(t+2\tau) = q_1(t+\tau) + q_2(t+\tau)$$

$$C_1 v_2(t+2\tau) + C_2 v_2(t+2\tau) =$$

$$= C_1 v_1(t+\tau) + C_2 v_2(t+\tau) = (C_1 + C_2) v_2(t+2\tau)$$

At time $t + 3\tau$. The charge at C_2 is conserved.

$$v_2(t+3\tau) = v_2(t+2\tau)$$

This is concluded into

$$(C_1 + C_2) v_2(t+3\tau) - C_2 v_2(t+\tau) = C_1 v_1(t+\tau)$$

Let $t = kT$ and $T = 2\tau$, z-transform

$$(z^{3/2}(C_1 + C_2) - z^{1/2} C_2) V_2(z) = C_1 z^{1/2} V_1(z)$$

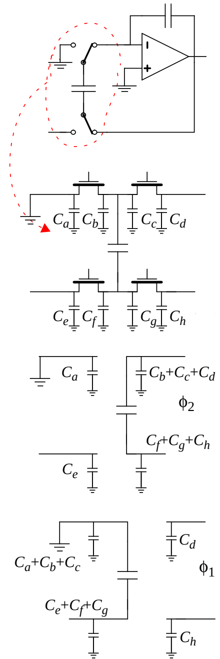
This gives the transfer function

$$H(z) = \frac{V_2(z)}{V_1(z)} = \frac{C_1}{z(C_1 + C_2) - C_2} = \frac{C_1/(C_1 + C_2)}{z - C_2/(C_1 + C_2)}$$

C_1 must be much larger than C_2, $C_1 » C_2$, to achieve a sample-and-hold cirucit

$$H(z) = z^{-1}$$

Example parasitics II



Consider the parasitic capacitances C_a through C_h.

During clock phase ϕ_2, C_b, C_c and C_d are coupled in parallel. The same is true for C_f, C_g and C_h. C_a is short and C_e is connected to the input signal.

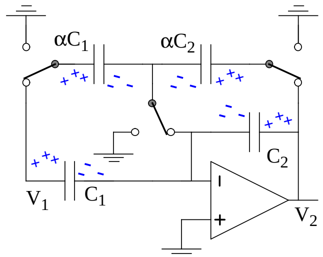
The charge on the parallel capacitances will redistribute to C_2.

During clock phase ϕ_1, C_a, C_b and C_c are coupled in parallel. The same is true for C_e, C_f and C_g. C_d is coupled to the input of the OPamp. C_h is connected to the output of the OPamp.

Now note that C_a, C_b, C_c and C_d always are connected to ground or virtual ground, hence always short and will not affect the transfer function. The charge on C_1 's plate connected to the input of the OPamp determines the transfer function. While the input signal is directly connected to C_1 neither will the capacitances C_e, C_f, C_g, or C_h affect the transfer function.

Exercises

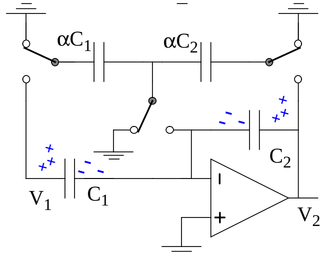
Exercise K36



Derive the transfer function $H(z)$.

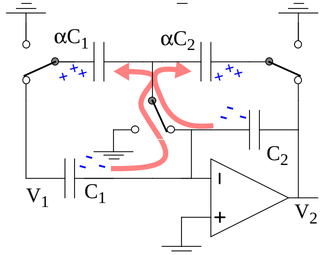
At time t the charge at the transistors is written as

$$q_1(t) = C_1 v_1(t)$$
$$q_2(t) = C_2 v_2(t)$$
$$q_{\alpha 1}(t) = \alpha C_1 v_1(t)$$
$$q_{\alpha 2}(t) = \alpha C_2 v_2(t)$$



At $t+\tau$, αC_1 and αC_2 are completely shorted. The total charge on C_1 and C_2 must however be conserved, while no charge can disappear from the input of the OPamp. Changes of the input signal will determine how the charge is distributed between C_1 and C_2:

$$q_1(t+\tau) + q_2(t+\tau) = q_1(t) + q_2(t)$$
$$q_{\alpha 1}(t+\tau) = q_{\alpha 2}(t+\tau) = 0$$



At $t+2\tau$ we use the same result. No charge disappears from the OPamp input. It has to redistribute to the other (previously discharged) capacitances:

$$q_1(t+\tau) + q_2(t+\tau) =$$
$$= q_1(t+2\tau) + q_2(t+2\tau) +$$
$$+ q_{\alpha 1}(t+2\tau) + q_{\alpha 2}(t+2\tau)$$
$$= q_1(t) - q_2(t)$$

This gives

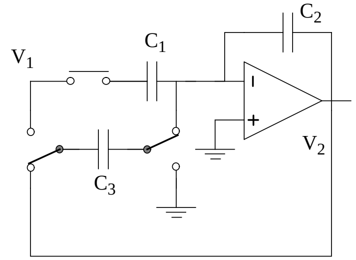
$$C_1 v_1(t) + C_2 v_2(t) = (1+\alpha) C_1 v_1(t+2\tau) + (1+\alpha) C_2 v_2(t+2\tau)$$

Let $t = kT$ and $2\tau = T$. z-transform and the transfer function is

$$H(z) = \frac{V_2(z)}{V_1(z)} = -\frac{C_1}{C_2} \cdot \frac{(1+\alpha)z - 1}{(1+\alpha)z - 1} = -\frac{C_1}{C_2} \cdot \frac{z - \frac{1}{1+\alpha}}{z - \frac{1}{1+\alpha}} = -\frac{C_1}{C_2}$$

which is an inverting amplifier. The pole is cancelled by the zero.

Exercise K27



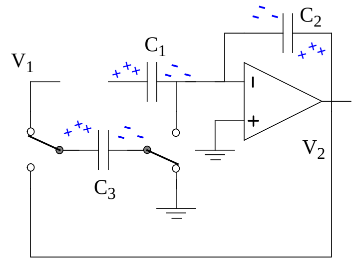
At time t the charge is discribed by

$$q_1(t) = C_1 v_1(t)$$
$$q_2(t) = C_2 v_2(t)$$
$$q_3(t) = C_3 v_2(t)$$

At time $t+\tau$:

Charge at C_1 and C_2

$$q_1(t+\tau) = q_1(t)$$
$$q_2(t+\tau) = q_2(t)$$



C_3 is charged with the input voltage

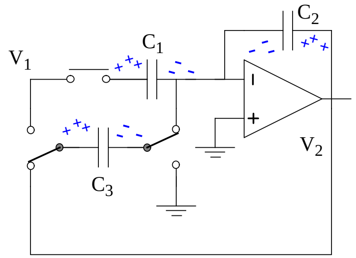
$$q_3(t+\tau) = C_3 v_1(t+\tau)$$

At time $t+2\tau$:

Total charge on the three capacitances is

$$q_1(t+2\tau) + q_2(t+2\tau) + q_3(t+2\tau)$$

where



$$q_1(t+2\tau) = C_1 v_1(t+2\tau)$$
$$q_2(t+2\tau) = C_2 v_2(t+2\tau)$$
$$q_3(t+2\tau) = C_3 v_2(t+2\tau)$$

The total charge must be conserved, no charge disappears from the input of the OPamp:

$$q_1(t+2\tau) + q_2(t+2\tau) + q_3(t+2\tau) =$$
$$= q_1(t+\tau) + q_2(t+\tau) + q_3(t+\tau) =$$
$$= q_1(t) + q_2(t) + q_3(t+\tau)$$

Use the charge expression, and we have

$$(C_2 + C_3) v_2(t+2\tau) + C_1 v_1(t+2\tau) = C_1 v_1(t) + C_2 v_2(t) + C_3 v_1(t+\tau)$$

which gives

$$v_2(t+2\tau) - \frac{C_2}{C_2+C_3} v_2(t) = \frac{C_1}{C_2+C_3}\left[v_1(t) + \frac{C_3}{C_1} v_1(t+\tau) - v_1(t+2\tau)\right]$$

Let $t = kT$ and $T = 2\tau$. z-transform

$$H(z) = \frac{V_2(z)}{V_1(z)} = \frac{C_1}{C_2+C_3} \frac{1 + \frac{C_3}{C_1} z^{1/2} - z}{z - \frac{C_2}{C_2+C_3}} = -\frac{C_1}{C_2+C_3} \frac{z - \left(1 + \frac{C_3}{C_1} z^{1/2}\right)}{z - \frac{C_2}{C_2+C_3}}$$

We now see that the output signal is affected by the input signal at each half clock period. Two ways can be used to design a first-order all pass filter.

1) Eliminate $z^{1/2}$ by assuming $v_1(t) = v_1(t+\tau)$ which gives $z^{1/2}V_1(z) = V_1(z)$

2) Eliminate $z^{1/2}$ by assuming $v_1(t+\tau) = v_1(t+2\tau)$ which gives $z^{1/2}V_1(z) = zV_1(z)$

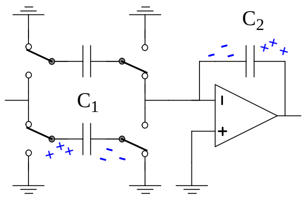
this gives

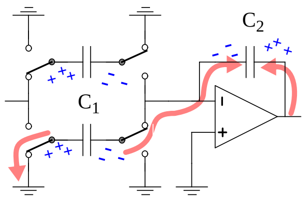
$$H_1(z) = -\frac{C_1}{C_2+C_3}\cdot\frac{z-\dfrac{C_1+C_3}{C_1}}{z-\dfrac{C_2}{C_2+C_3}} \text{ or } H_2(z) = -\frac{C_1}{C_2+C_3}\cdot\left(1-\frac{C_3}{C_1}\right)\frac{z-\dfrac{1}{1-C_3/C_1}}{z-\dfrac{C_2}{C_2+C_3}}$$

For an all pass filter, if the pole is given by $z = p$, the zero is given by $z = 1/p$. This gives

$$\frac{C_1+C_3}{C_1} = \frac{C_2+C_3}{C_2} \Rightarrow C_2 = C_1 \text{ or } 1-\frac{C_3}{C_1} = \frac{C_2}{C_2+C_3} \Rightarrow C_1 = C_2+C_3$$

Exercise K28





At time t the lower C_1 is charged

$$q_{N1}(t) = C_1v_1(t)$$

The upper is shorted.

$$q_{U1}(t) = 0$$

At time $t+\tau$ the upper C_1 is charged

$$q_{U1}(t+\tau) = C_1v_1(t+\tau)$$

The lower is shorted. The charge will however redistribute to the negative plate at C_2. The positive plate at C_2 will get extra charge from the output of the OPamp. The charge at C_2 is written as

$$q_2(t+\tau) = C_2v_2(t+\tau) = q_2(t)+q_{N1}(t) =$$
$$= C_2v_2(t)+C_1v_1(t)$$

At time $t+2\tau$ the operation is practical the same due to the symmetrical capacitances.

$$q_2(t+2\tau) = C_2v_2(t+2\tau) = q_2(t+\tau)+q_{U1}(t+\tau) = C_2v_2(t+\tau)+C_1v_1(t+\tau)$$

We see that the input signal is delayed and switched to the output at every half clock cycle. We have

$$v_2(t+2\tau) = v_2(t)+\frac{C_1}{C_2}[v_1(t+\tau)+v_1(t)]$$

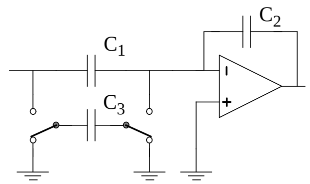
And the transfer function is

$$H(z) = \frac{C_1}{C_2}\cdot\frac{z^{1/2}+1}{z-1}$$

If we now once again assume that $v_1(t) = v_1(t+\tau)$ or $v_1(t+\tau) = v_1(t+2\tau)$, then

$$H(z) = \frac{C_1}{C_2}\cdot\frac{2z^{-1}}{1-z^{-1}} \text{ or } H(z) = \frac{C_1}{C_2}\cdot\frac{1+z^{-1}}{1-z^{-1}}$$

Exercise K38

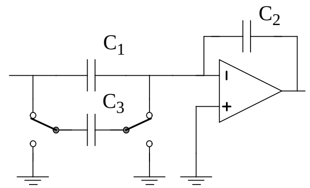


General transfer function for bilinear integrator:

$$H(z) = K\cdot\frac{z-1}{z+1}$$

At time t the charge distribution is

$$q_1(t) = C_1v_1(t)$$
$$q_2(t) = C_2v_2(t)$$
$$q_3(t) = 0 \text{ (shorted)}$$



At time $t+\tau$ C_3 is coupled in parallel with C_1 and will take charge from C_2 and C_1:

$$q_1(t+\tau) = C_1v_1(t+\tau)$$
$$q_2(t+\tau) = C_2v_2(t+\tau)$$
$$q_3(t+\tau) = C_3v_1(t+\tau)$$

The charge distribution will be

$$q_1(t+\tau)+q_2(t+\tau)+q_3(t+\tau) = q_1(t)+q_2(t)$$

At time $t+2\tau$ the total charge at C_1 and C_2 is conserved. It will though redistribute due to the change of input voltage.

$$q_1(t+\tau)+q_2(t+\tau) = q_1(t+2\tau)+q_2(t+2\tau)$$

Concludingly, we have

$$q_1(t)+q_2(t) = q_3(t+\tau)+q_1(t+2\tau)+q_2(t+2\tau)\text{, i.e.,}$$
$$C_1v_1(t)-C_1v_1(t+2\tau)-C_3v_1(t+\tau) = C_2[v_2(t+2\tau)-v_2(t)]$$

which gives

$$v_2(t+2\tau)-v_2(t) = -\frac{C_1}{C_2}\left[v_1(t+2\tau)+\frac{C_3}{C_1}v_1(t+\tau)-v_1(t)\right]$$

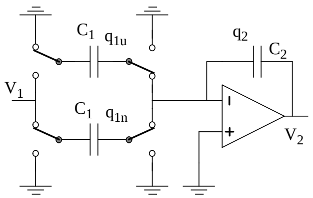
Suppose $v_1(t) = v_1(t+\tau)$, hence a sample-and-hold circuit at the input, which eliminated the $z^{1/2}$-term in the transfer function. Let $t = kT$ and $2\tau = T$. z-transform

$$H(z) = \frac{V_2(z)}{V_1(z)} = -\frac{C_1}{C_2}\cdot\frac{z+(C_3/C_1-1)}{z-1}$$

Choosee $C_3 = 2C_1$ and we have

$$H(z) = -\frac{C_1}{C_2} \cdot \frac{1+z^{-1}}{1-z^{-1}}$$

Exercise K37



At time t. The upper capacitor is shorted between ground and virtual ground and the lower capacitor is charged with the input voltage:

$$q_{1u}(t) = 0 \text{ and } q_{1n}(t) = C_1 v_1(t)$$

C_2 has the charge:

$$q_2(t) = C_2 v_2(t)$$

At time $t + \tau$. The upper capacitor is charged.

$$q_{1u}(t+\tau) = C_1 v_1(t+\tau)$$

The lower capacitor is shorted, all charge is lost to the ground.

$$q_{1n}(t+\tau) = 0$$

The charge in C_2 is conserved since no charge can disappear from the input of the OPamp.

$$q_2(t+\tau) = q_2(t) \text{ dvs } C_2 v_2(t+\tau) = C_2 v_2(t) \text{ dvs } v_2(t+\tau) = v_2(t)$$

Time $t + 2\tau$. The upper capacitor is discharged, but its charge will be redistributed over the lower capacitor and C_2. The redistribution is determined by the input voltage. We have

$$q_{1u}(t+2\tau) = 0,\ q_{1n}(t+2\tau) = C_1 v_1(t+2\tau),\ q_2(t+2\tau) = C_2 v_2(t+2\tau)$$

and

$$-q_{1n}(t+2\tau) + (-q_2(t+2\tau)) = -q_{1u}(t+\tau) + (-q_2(t+\tau))$$

Which gives

$$C_1 v_1(t+2\tau) + C_2 v_2(t+2\tau) = C_1 v_1(t+\tau) + C_2 v_2(t+\tau) = C_1 v_1(t+\tau) + C_2 v_2(t)$$

The input signal is sampled-and-held as

$$v_1(t+\tau) = v_1(t)$$

which gives

$$C_1 v_1(t+2\tau) + C_2 v_2(t+2\tau) = C_1 v_1(t) + C_2 v_2(t)$$

z-transform

$$C_1 \cdot (z-1) \cdot V_1(z) = C_2 \cdot (1-z) \cdot V_2(z)$$

and

$$H(z) = \frac{V_1(z)}{V_2(z)} = -\frac{C_2}{C_1} \cdot \frac{z-1}{z-1} = -\frac{C_2}{C_1}$$

The circuit is an inverting amplifier. Practically, however, a pole on the unit circle can not be

cancelled by a zero. The circuit has to be used in a feedback loop.

Exercise B10.2

At time t, switch ϕ_1 is conducting. The charges at C_1 and C_2 are given by

$$q_1(t) = C_1 \cdot v_1(t) \text{ and } q_2(t) = C_2 \cdot v_2(t)$$

At time $t + \tau$, switch ϕ_2 is conducting. The charges at C_1 and C_2 are given by

$$q_1(t+\tau) = 0 \text{ and } q_2(t+\tau) = C_2 \cdot v_2(t+\tau)$$

Note that the positive and negative plates of C_1 are connected. The charge will cancel themselves, therefore

$$q_2(t+\tau) = q_2(t)$$

At time $t + 2\tau$, switch ϕ_1 is conducting. Now we will have a redistribution of the charge at the negative plate of C_2 (the one connected to virtual ground at the OPamp input).

$$q_1(t+2\tau) = C_1 \cdot v_1(t+2\tau),\ q_2(t+2\tau) = C_2 \cdot v_2(t+2\tau) \text{ and}$$
$$-q_1(t+2\tau) + (-q_2(t+2\tau)) = -q_2(t+\tau) = -q_2(t)$$

This gives

$$C_1 \cdot v_1(t+T) + C_2 \cdot v_2(t+T) = C_2 \cdot v_2(t)$$

where $T = 2\tau$. z-transforming the equation gives

$$\frac{V_2(z)}{V_1(z)} = H(z) = \frac{C_1}{C_2} \cdot \frac{z}{1-z} = -\frac{C_1}{C_2} \cdot \frac{1}{1-z^{-1}}$$

which is a invering and scaling integrator.

Exercises B10.3 - 4 are very suitable for calculation.