Lecture 6, ATIK

Switched-capacitor circuits 2
S/H, Some nonideal effects
Continuous-time filters

What did we do last time?

Switched capacitor circuits

- The basics
- Charge-redistribution analysis

Nonidealties

- SC parasitics

What will we do today?

Switched capacitor circuits with non-ideal effects in mind

- What should we look out for?
- What is the impact on system performance, like filters.

Continuous-time filters

- Active-RC
- Transconductance-C
- Second-order links
- Leapfrog filters

Mainly overview

A list of non-ideal effects in SC

System

- Parasitics
- Noise

OP

- Offset error
- Gain
- Bandwidth (output impedance)
- Slew rate
- Noise

Switches

- On-resistance
- Clock feed through, Charge injection
- Jitter

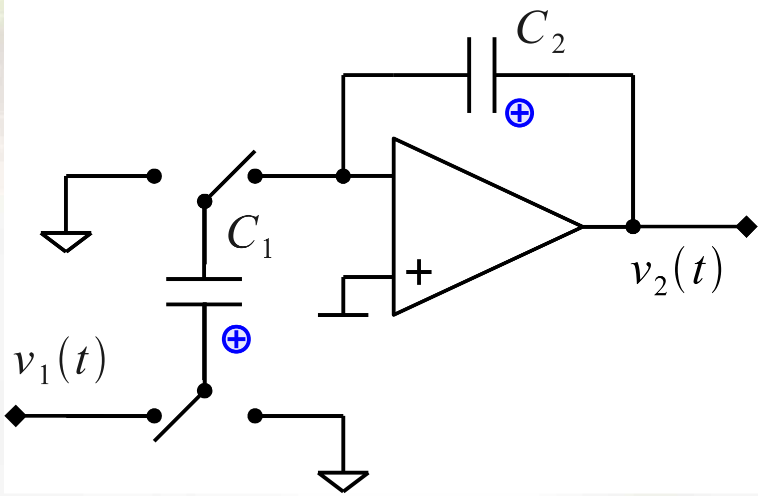
An example SC accumulator to work on

Phase 1:

$$q_1(nT) = C_1 \cdot V_1(nT), \quad q_2(nT) = C_2 \cdot V_2(nT)$$

Phase 2:

$$q_1(nT+\tau) = 0, \quad q_2(nT+\tau) = C_2 \cdot V_2(nT+\tau)$$



Charge preservation:

$$q_2(nT+\tau) = q_2(nT) \Rightarrow V_2(nT+\tau) = V_2(nT)$$

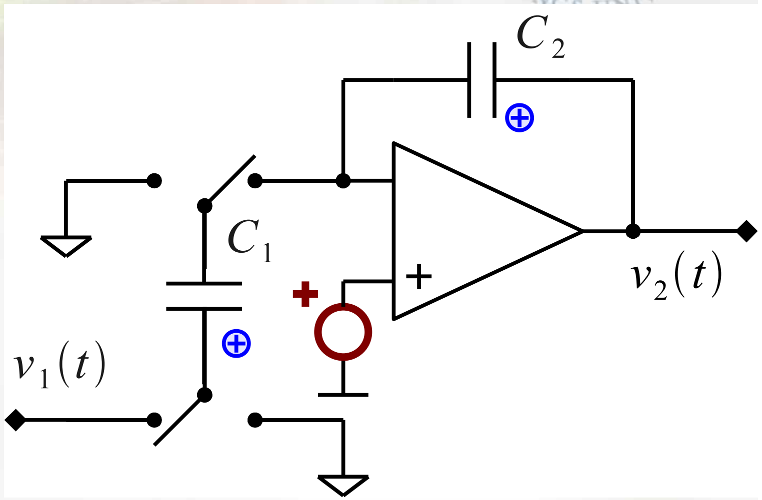
$$-q_1(nT) - q_2(nT) = -q_1(nT-\tau) - q_2(nT-\tau) = -q_2(nT-T)$$

Laplace transform

$$C_1 \cdot V_1(z) = -C_2 \cdot \left(1 - z^{-1}\right) \cdot V_2(z) \Rightarrow \frac{V_2(z)}{V_1(z)} = -\frac{C_1 / C_2}{1 - z^{-1}}$$

Impact of offset error

Due to mismatch in differential pair, we will have an offset at the input of our amplifier



$$I = (\alpha + \Delta\alpha) \cdot V_{eff}^2 \Rightarrow \Delta I = \Delta\alpha \cdot V_{eff}^2$$

The offset error is in the order of

$$\Delta V_{eff} = \frac{\Delta I}{g_m} = \frac{\Delta\alpha \cdot V_{eff}^2}{\frac{2 I_D}{V_{eff}}} = \frac{\Delta\alpha}{\alpha} \cdot \frac{V_{eff}}{2}$$

With the help of gain, we can propagate any mismatch back to the input which results in a constant voltage, v_x, on (one of) the input(s)

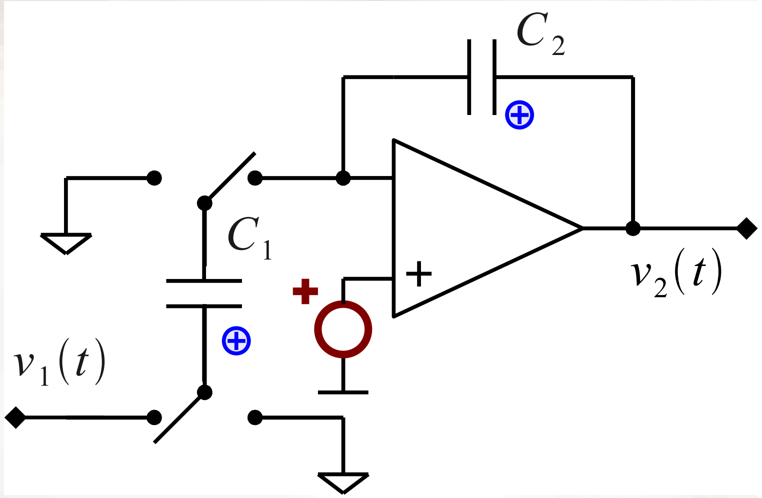
Impact of offset error, cont'd

Phase 1

$$q_1(nT)=C_1\cdot(v_1(nT)-v_x),\quad q_2(nT)=C_2\cdot(v_2(nT)-v_x)$$

Phase 2

$$q_1(nT+\tau)=0,\quad q_2(nT+\tau)=C_2\cdot(v_2(nT+\tau)-v_x)$$



Charge preservation

$$-q_1(nT)-q_2(nT)=-q_1(nT+\tau)-q_2(nT+\tau)$$

$$q_2(nT+\tau)=q_2(nT+T)\Rightarrow v_2(nT+\tau)=v_2(nT+T)$$

$$C_1v_1(nT)-C_1v_x+C_2v_2(nT)-C_2v_x=C_2v_2(nT+T)-C_2v_x$$

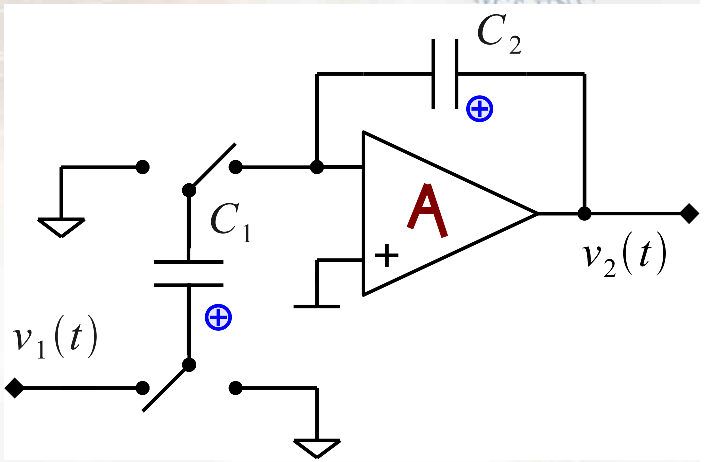
$$C_1V_1(z)-\underbrace{\frac{C_1v_x}{1-z^{-1}}}_{C_1v_x}+C_2V_2(z)=C_2V_2(z)\cdot z\Rightarrow V_2(z)=\frac{-C_1/C_2}{1-z^{-1}}\cdot V_1(z)-\underbrace{\frac{v_x\cdot C_1/C_2}{(1-z^{-1})^2}}_{\text{ooops!}}$$

Output is ramped due to offset!

Impact of gain errors

Phase 1

$$q_1(nT)=C_1\cdot\left(v_1(nT)+\frac{v_2(nT)}{A_0}\right),\quad q_2(nT)=C_2\cdot\left(v_2(nT)+\frac{v_2(nT)}{A_0}\right)$$



Phase 2

$$q_1(nT+\tau)=0,\quad q_2(nT+\tau)=C_2\cdot\left(v_2(nT+\tau)+\frac{v_2(nT+\tau)}{A_0}\right)$$

Charge preservation

$$-q_1(nT)-q_2(nT)=-q_1(nT+\tau)-q_2(nT+\tau),$$

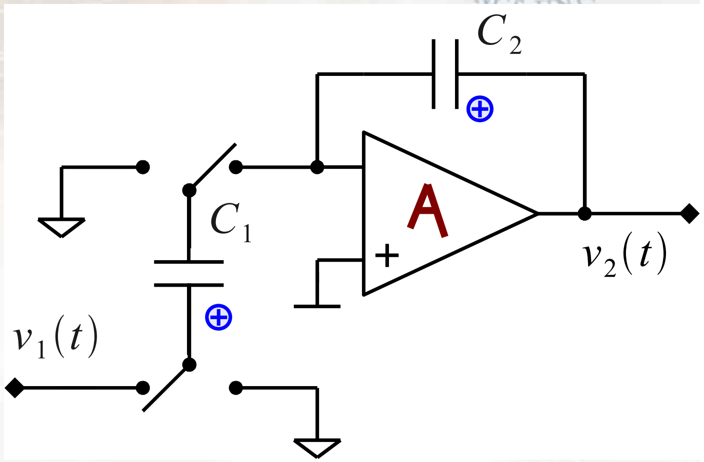
$$q_2(nT+\tau)=q_2(nT+T)\Rightarrow v_2(nT+\tau)=v_2(nT+T)$$

$$C_1v_1(nT)=-v_2(nT)\cdot\left(\frac{C_1}{A_0}+C_2\cdot\left(1+\frac{1}{A_0}\right)\right)+C_2\left(1+\frac{1}{A_0}\right)\cdot v_2(nT+T)$$

Impact of gain errors, cont'd

Compiled

$$\frac{V_2(z)}{V_1(z)} = \frac{\dfrac{C_1/C_2}{1+1/A_0}}{z-\left(1+\dfrac{C_1/C_2}{A_0+1}\right)^{-1}}$$

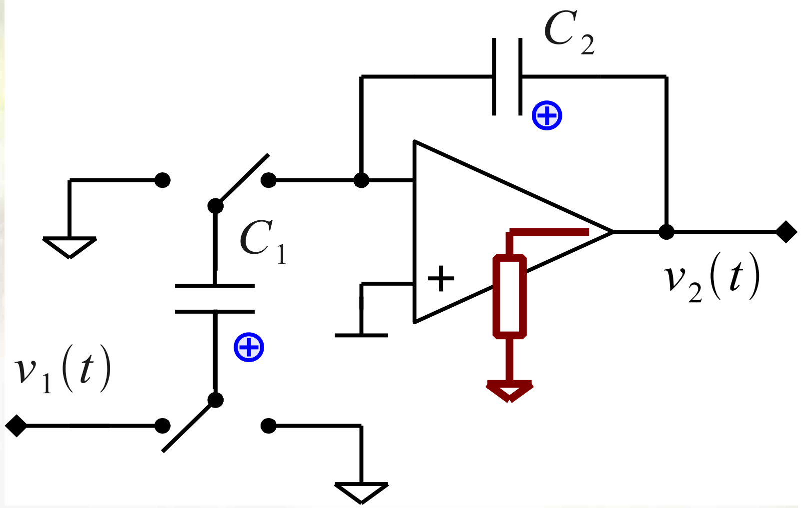


Introduces a gain error and a pole shift (!)

Lossy integrator, i.e., a low-pass filter with a DC gain of

$$\frac{V_2(1)}{V_1(1)} = \frac{\dfrac{1}{1+1/A_0}}{\dfrac{1}{A_0+1}} = A_0 \neq \infty$$

Impact of bandwidth



The speed (bandwidth) of the OP is given by the unity-gain frequency and feedback factor

$$H(s)=\frac{1/\beta}{1+\frac{1}{A(s)\cdot\beta}}=\frac{1/\beta}{1+\frac{1+s/p_1}{A_0\cdot\beta}}$$

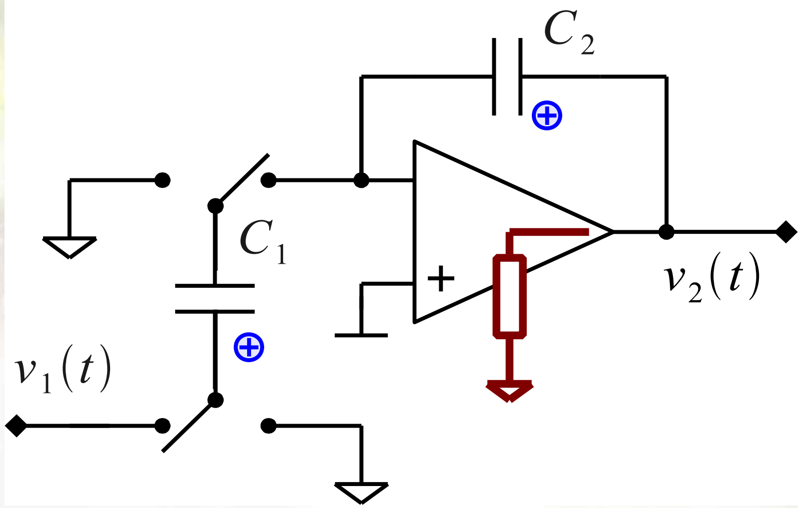
$$\approx\frac{1/\beta}{1+\frac{s}{\beta\cdot A_0\cdot p_1}}\approx\frac{1/\beta}{1+\frac{s}{\beta\cdot\omega_{ug}}}$$

This means that the output will follow a step response according to:

$$v_2(nT+t)=v_2(nT)+\Delta V_2\cdot\left(1-e^{-t\beta\omega_{ug}}\right)$$

Notice that the feedback factor varies with different phases!

Impact of bandwidth, cont'd



Phase 2: discharging C_1 is instantaneous. V_2 cannot change either, since charge on C_2 is maintained due to the infinitely fast switch.

Phase 1: we re-charge C_1 and settling of V_2 will determine how fast we can do that.

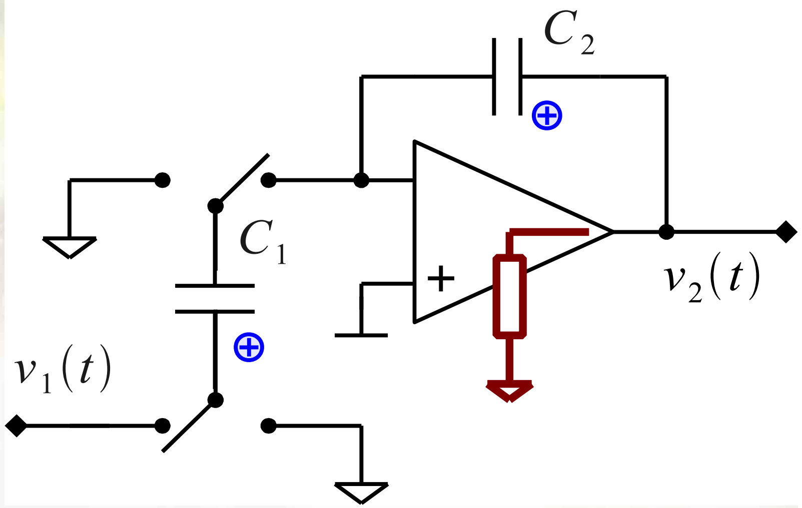
A first-order, lazy approximation:

$$v_2(nT+\tau)=v_2(nT)+\frac{C_1}{C_2}\cdot v_1(nT)$$ (ideal)

$$v_2(nT+t)=v_2(nT)+\left(\underbrace{v_2(nT)+\frac{C_1}{C_2}\cdot v_1(nT)}_{v_2(nT+\tau)}-v_2(nT)\right)\cdot\left(1-e^{-t\beta\omega_{ug}}\right)$$ (actual)

$$v_2(nT+\tau)=v_2(nT)+\frac{C_1}{C_2}\cdot v_1(nT)\cdot\underbrace{\left(1-e^{-\tau\beta\omega_{ug}}\right)}_{B}$$

Impact of bandwidth, cont'd



Compiled (notice the approximation, in reality an additional time-shifted gain component too!):

$$\frac{V_2(z)}{V_1(z)} \approx B \cdot \underbrace{\frac{C_1/C_2}{z-1}}_{\text{Ideal}}$$

“Less” of a problem in this case, in a first-order analysis, it results in a gain error.

The ideal-switch assumption and charge preservation forces the accumulation to not be lossy.

The clock frequency, f_s, is hidden in the equation

and if $\omega_{ug} \approx 2\pi f_s$, the B is a rather small value!

Impact of slew rate

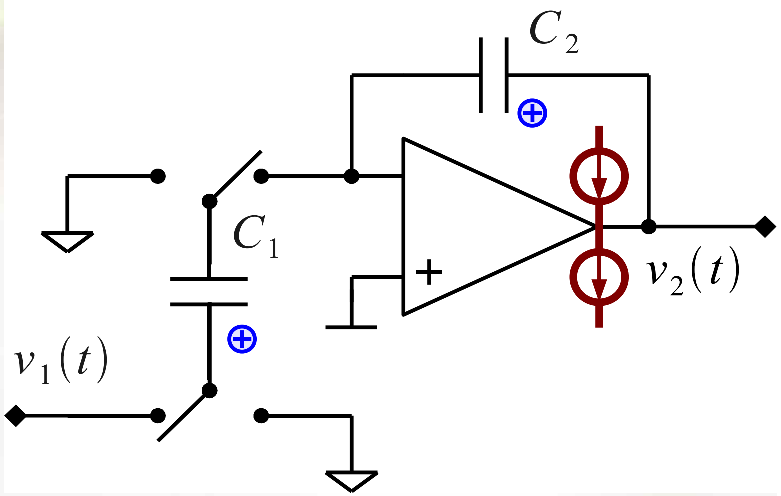
Slew rate is a non-linear function.

The output will now follow:

$$v_2(nT+t)=v_2(nT)+\frac{I_{max}}{C_L}\cdot t$$

Within a phase, we are able to reach

$$\Delta v_{2,max}=\frac{I_{max}}{C_L}\cdot\tau$$



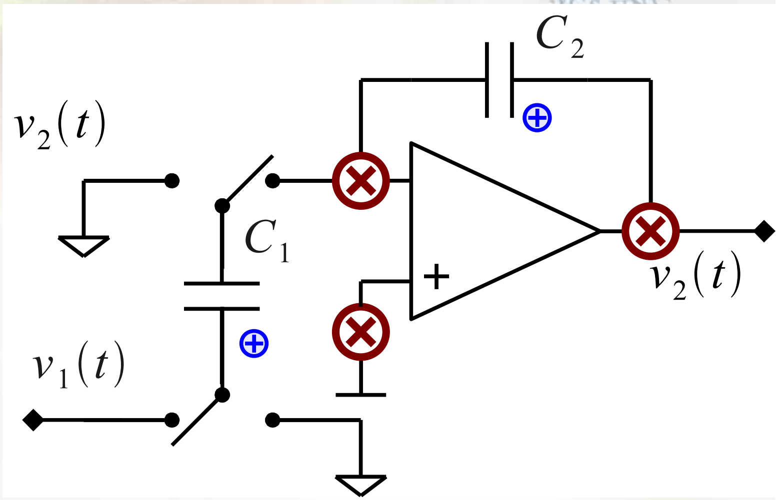
Generic analysis hard, since only large voltage steps are affected

If SR is not avoided we have a highly distorted signal!

Impact of noise

Noise due to a switch in Nyquist band:

$$v_C^2(f)=4kT\cdot R_{\text{equiv}}=\frac{4kT}{f_s\cdot C_1}$$



Noise from operational amplifier:

Given by input-referred noise voltage:

$$v_{op}^2(f)$$

Noise is sampled, i.e., aliased

At the sampling instant, the noise voltage at the input of the OP is sampled.

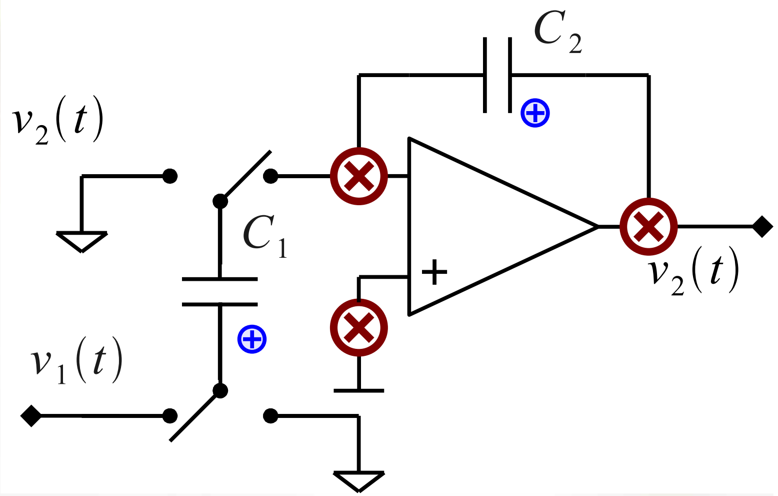
C.f. offset error

$$\ldots+\frac{C_1/C_2}{1-z}\cdot V_X(z)$$

Impact of noise, cont'd

The super function

$$S_{out}(z)=|H(z)|^2\cdot S_{in}(z)\Rightarrow\left|\frac{-C_1/C_2}{1-z}\right|^2\cdot S_{in}(z)$$

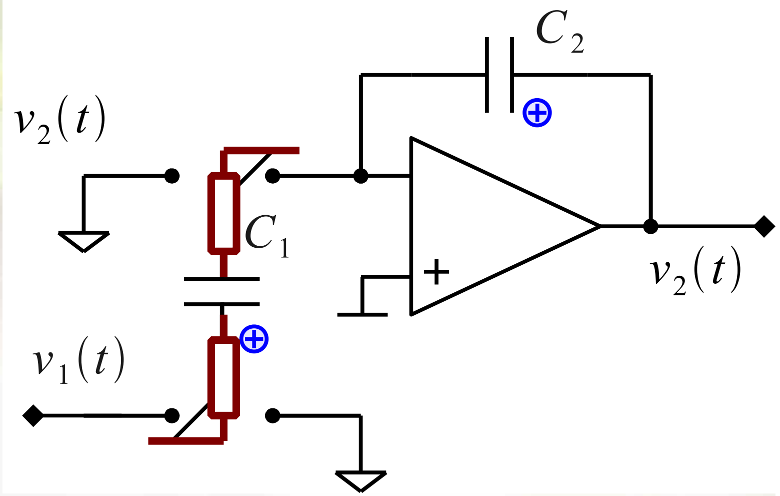


The transfer function (in this case) modifies the noise to the output and noise is integrated too.

Beware! Continuous-time noise vs sampled noise.

(When do you “measure” the noise)?

Impact of on-resistance



Similar to limited bandwidth, but now a bit more accurate step-by-step approach:

$$q_1(\tau)=C_1\cdot v_1(\tau)\cdot\left(\underbrace{1-e^{-\frac{\tau}{RC_1}}}_{B}\right)=C_1\cdot v_1(\tau)\cdot B$$

$$q_1(T)=C_1\cdot\frac{q_1(\tau)}{C_1}\cdot\underbrace{e^{-\frac{\tau}{RC_1}}}_{A}=q_1(\tau)\cdot A$$

$$q_1(T+\tau)=C_1\cdot\frac{q_1(T)}{C_1}+C_1\cdot\left(v_1(T+\tau)-\frac{q_1(T)}{C_1}\right)\cdot B$$

$$q_1(T+\tau)=A\cdot q_1(\tau)+\left(C_1\cdot v_1(T+\tau)-q_1(\tau)\cdot A\right)\cdot B=A^2\cdot q_1(\tau)+C_1\cdot v_1(T+\tau)\cdot B$$

Laplace

$$Q_1(z)\cdot z^{0.5}=A^2\cdot Q_1(z)\cdot z^{-0.5}+C_1\cdot V_1(z)\cdot z^{0.5}\cdot B\Rightarrow Q_1(z)=\frac{C_1\cdot V_1(z)}{1-A^2\cdot z^{-1}}$$

Impact of on-resistance, cont'd

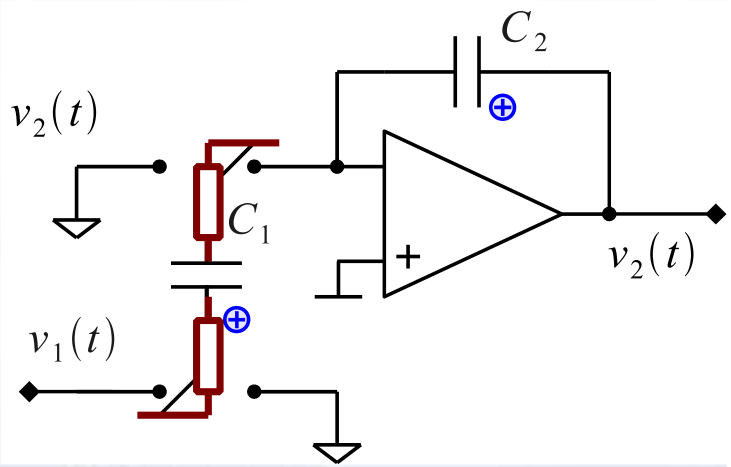
Charge accumulation and preservation must still hold

$$q_1(nT)+q_2(nT)=q_1(nT-\tau)+q_2(nT-\tau)$$

Be careful with which charge is which ...

$$A\cdot Q_1(z)\cdot z^{-0.5}+C_2\cdot V_2(z)\cdot z^{0.5}=Q_1(z)\cdot z^{-0.5}+C_2\cdot V_2(z)\cdot z^{-0.5}$$

$$(A-1)\cdot\frac{C_1\cdot V_1(z)}{1-A^2\cdot z^{-1}}=C_2\cdot V_2(z)\cdot(1-z)$$



Finally, we get that small time shift (additional pole, close to the origin)

$$\frac{V_2(z)}{V_1(z)}=\frac{B}{1-A^2\cdot z^{-1}}\cdot\underbrace{\frac{-C_1/C_2}{z-1}}_{\text{Ideal}}$$

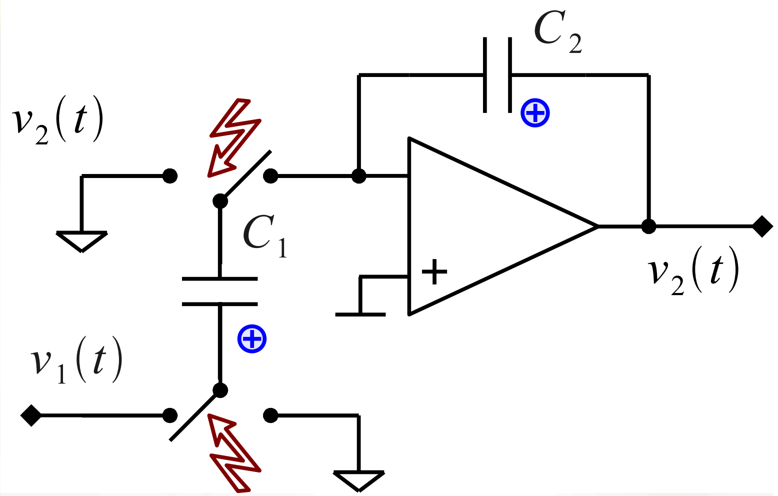
Impact of charge feed-through

Channel charge injection

$$q_{ch}(nT) \sim W\,L \cdot C_{ox} \cdot \left(V_x - V_1(nT)\right)$$

Clock feed-through

$$q_{CFT}(nT) = C_{ol} \cdot \left(V_y - V_1(nT)\right)$$



One signal-dependent part and one constant

Gain error $C_1' = C_1 + \Delta C$

Offset accumulation, $W\,L \cdot C_{ox} \cdot V_x + C_{ol} \cdot V_y$

The errors can be reduced to nearly zero using e.g. differential signals, switch dummies, and careful sizing.

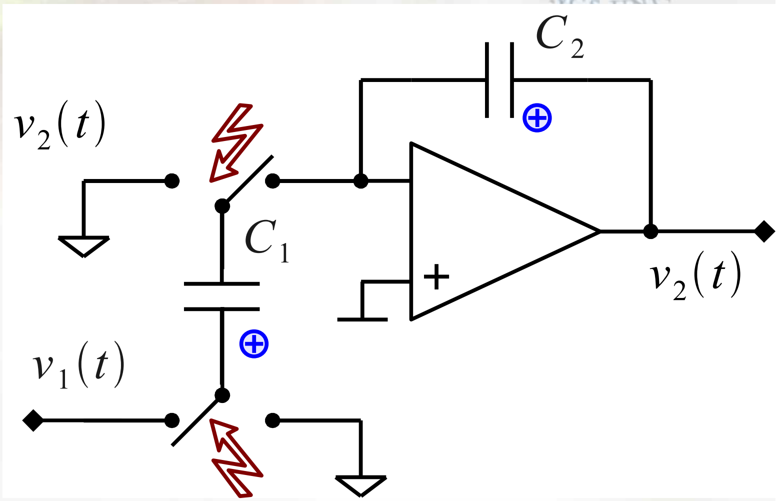
Impact of jitter

Sampling instant will be varying

$$nT' = nT + \delta T$$

where δT might be a stochastic and/or signal-dependent component

$v_1(t) = \sin(\omega t)$. i.e., $v_1(nT) = \sin(\omega nT)$



Taylor

$$v_1(nT + \delta T) \approx v_1(nT) + v_1(nT) \cdot \frac{\delta v_1(t)}{\delta t}$$

Example

$$v_1(nT + \delta T) = \sin \omega nT \cdot \cos \omega \delta T + \cos \omega nT \cdot \sin \omega \delta T \approx \sin \omega nT + \omega \cdot \delta T \cdot \cos \omega nT$$

The higher signal frequency the worse!

Conclusions SC building blocks

Many, many possible error sources

Today we've looked at different ways to model and address them and their different impacts

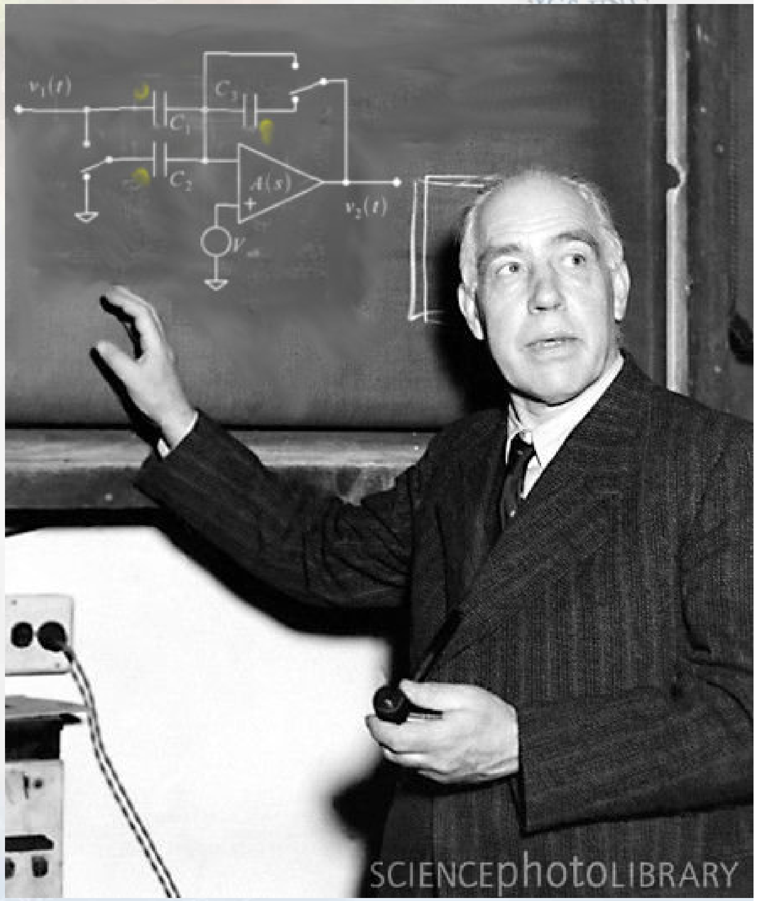
Notable error impacts

- Offset accumulation
- Overall gain error
- Shifted pole (lossy integrator)
- Parasitic pole (“nonlinear” integration)
- Nonlinearity (distortion)
- Jitter

Continuous-time filters

Filters and filtering functions are everywhere...

- Band-select
- Anti-aliasing
- Reconstruction filters



Continuous-time filters

A general transfer function for a linear system is given by

$$H(s)=\frac{Y(s)}{X(s)}=\frac{a_0+a_1 s+a_2 s^2+\dots}{b_0+b_1 s+b_2 s^2+\dots}$$

Rewrite in its original ODE form

$$Y(s)\cdot\left(b_0+b_1 s+b_2 s^2+\dots\right)=X(s)\cdot\left(a_0+a_1 s+a_2 s^2+\dots\right)$$

“Invert” to get integrations rather than derivations, and scale

$$Y(s)=X(s)\cdot\left(\alpha_0+\alpha_1\frac{1}{s}+\alpha_2\frac{1}{s^2}+\dots\right)-Y(s)\cdot\left(\beta_1\frac{1}{s}+\beta_2\frac{1}{s^2}+\dots\right)$$

Create a “recursive” set of integrations

$$Y(s)=\alpha_0\cdot X(s)+\frac{1}{s}\cdot\left(\alpha_1 X(s)-\beta_1\cdot Y(s)+\frac{1}{s}\cdot\left(\alpha_2\cdot X(s)-\beta_2\cdot Y(s)+\frac{1}{s}\cdot(\dots)\right)\right)$$

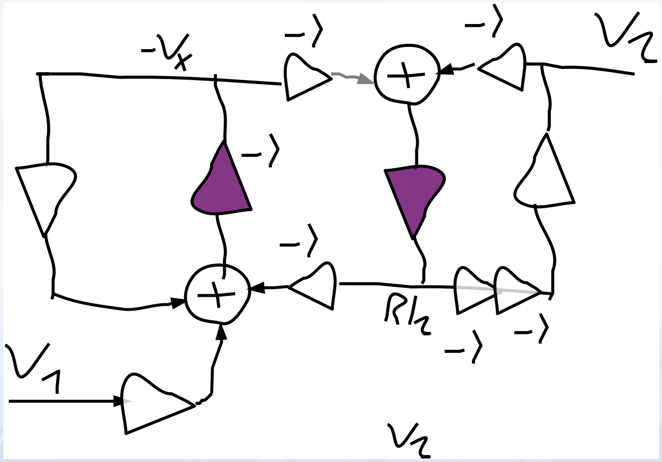
LiU EXPANDING REALITY

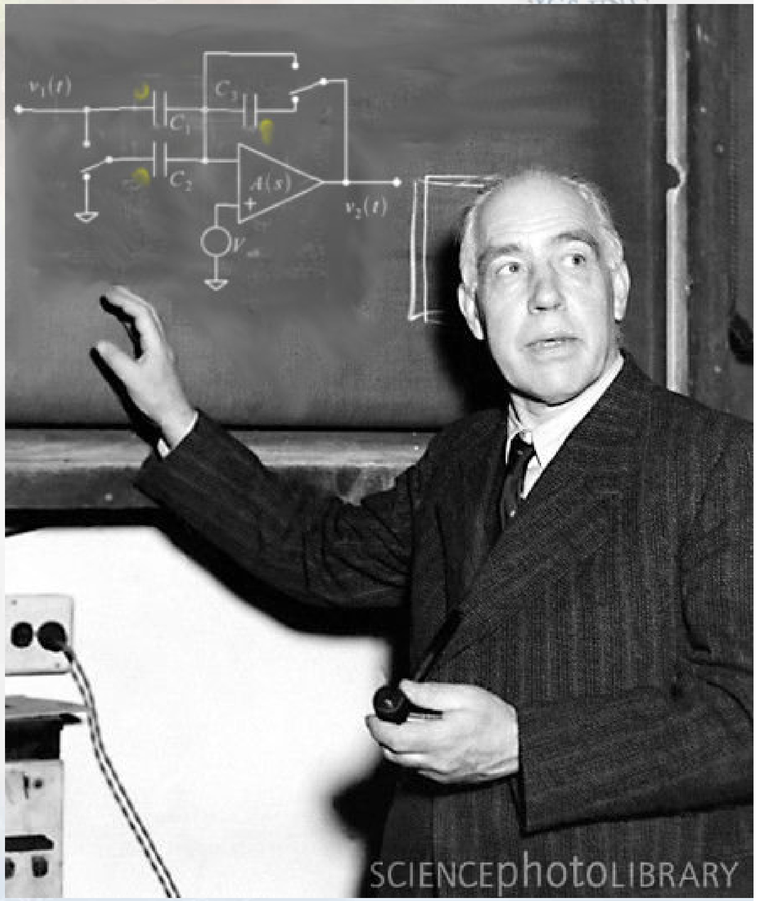
Continuous-time filters, flow graph

Manipulation

Replacing with integrators

Feedback, etc.





LiU EXPANDING REALITY

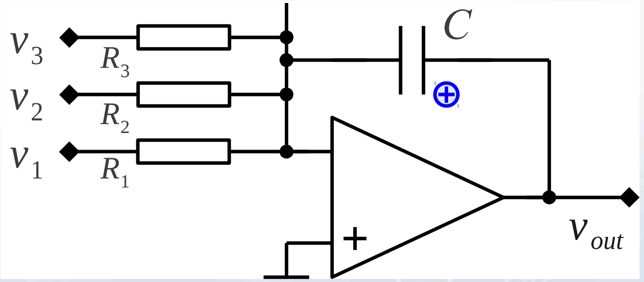
Active-RC

Summation of different inputs is done with the resistors, i.e., we are summing up the currents in the virtual ground node:

$$-V_{out}(s)\cdot s\,C_L = I_1(s) + I_2(s) + I_3(s) = \frac{V_1(s)}{R_1} + \frac{V_2(s)}{R_2} + \ldots$$

Which combined gives us the integration

$$V_{out}(s) = -\frac{V_1(s)}{s\,C_L R_1} - \frac{V_2(s)}{s\,C_L R_2} + \ldots$$



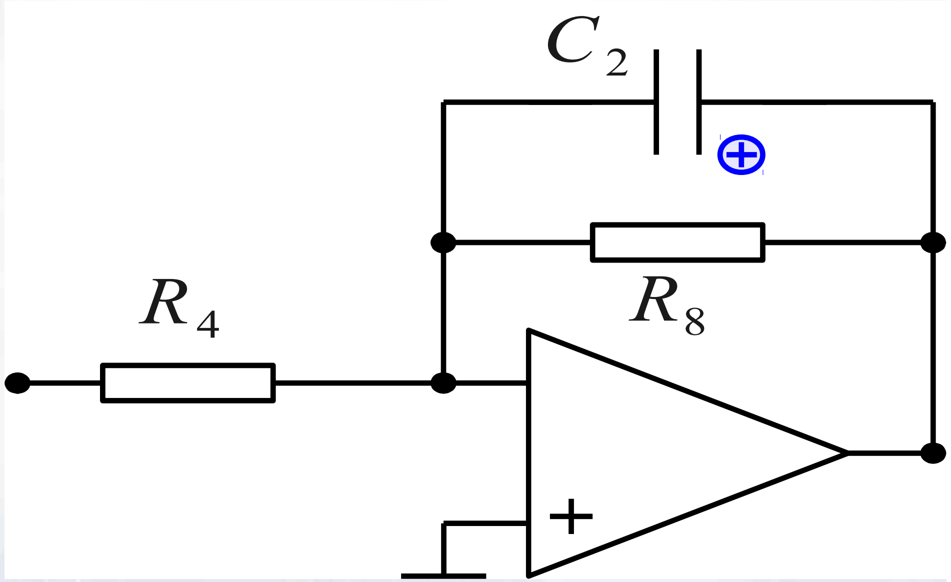
Example, first-order pole with active-RC

Sum the currents in the virtual ground:

$$V_{out}(s) \cdot s C_2 = \frac{-V_{in}(s)}{R_4} - \frac{V_{out}(s)}{R_8}$$

such that

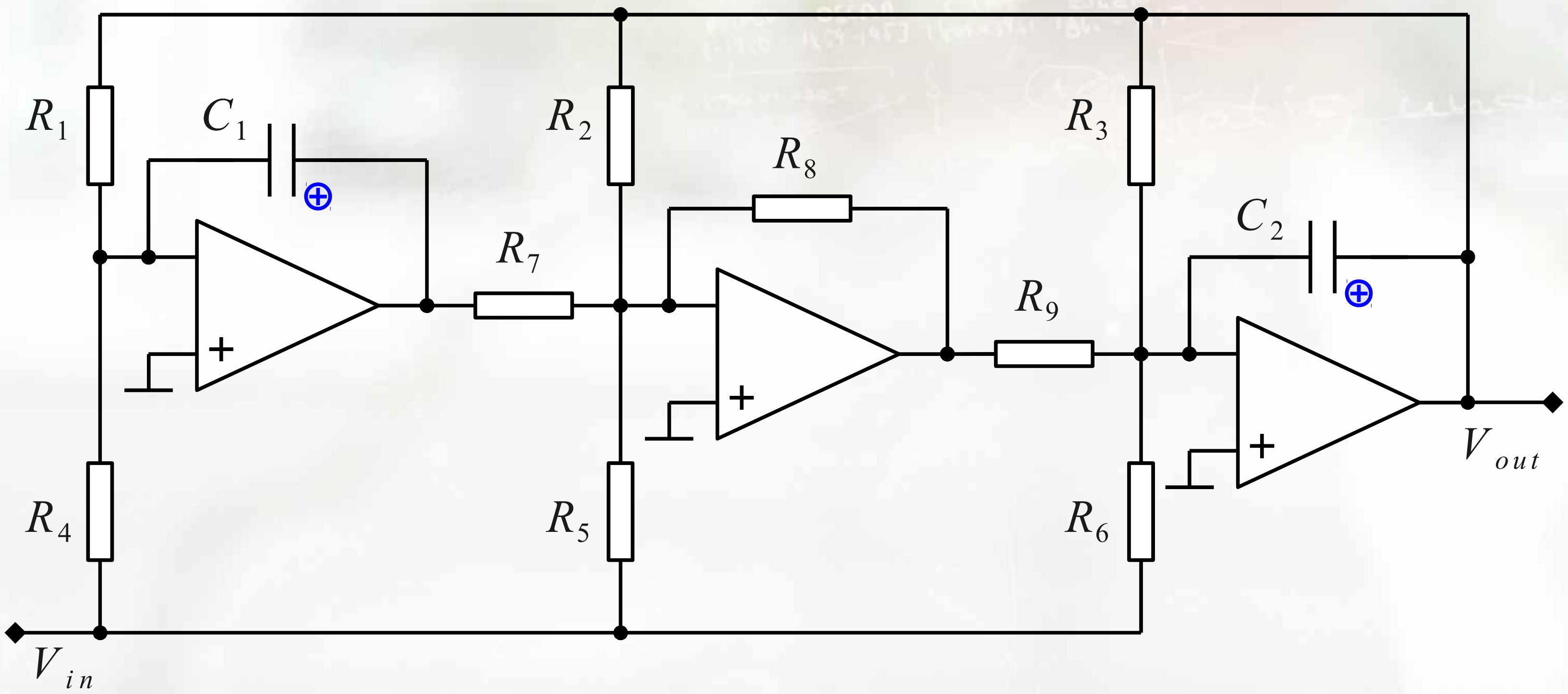
$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{-R_8/R_4}{1 + \frac{s}{1/C_2 R_4}}$$



LiU EXPANDING REALITY

Example: Tow-Thomas, Biquad, ...

Second-order link cascaded to form overall transfer function

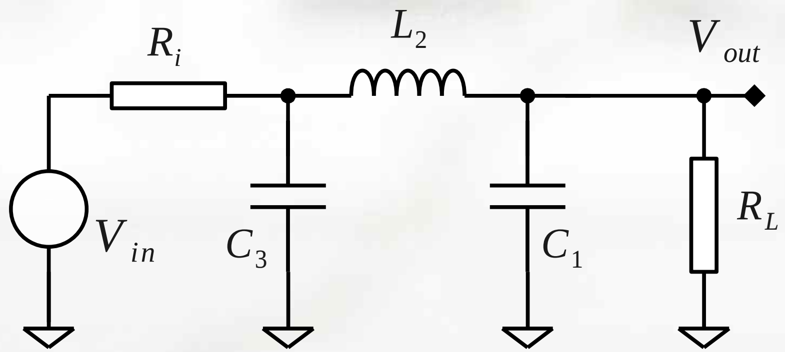


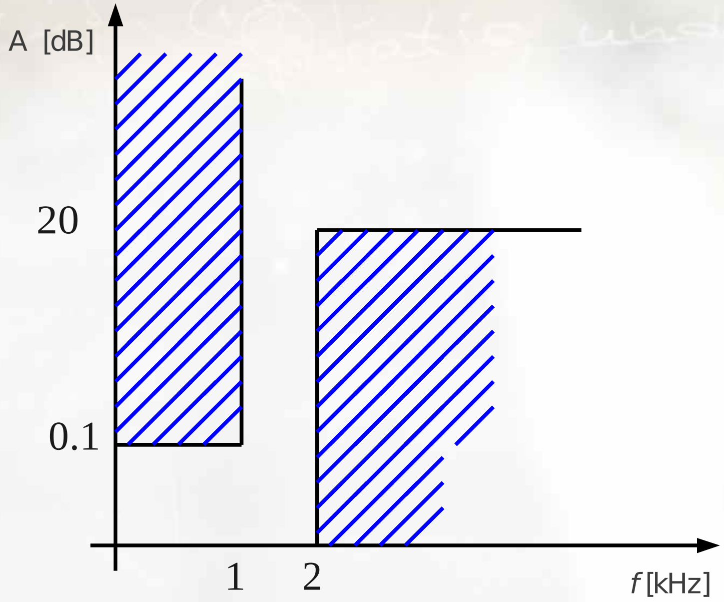
Output isolated and buffered with OP

Single-pole formed with single RC at output or input

Ladder networks

Use a reference filter

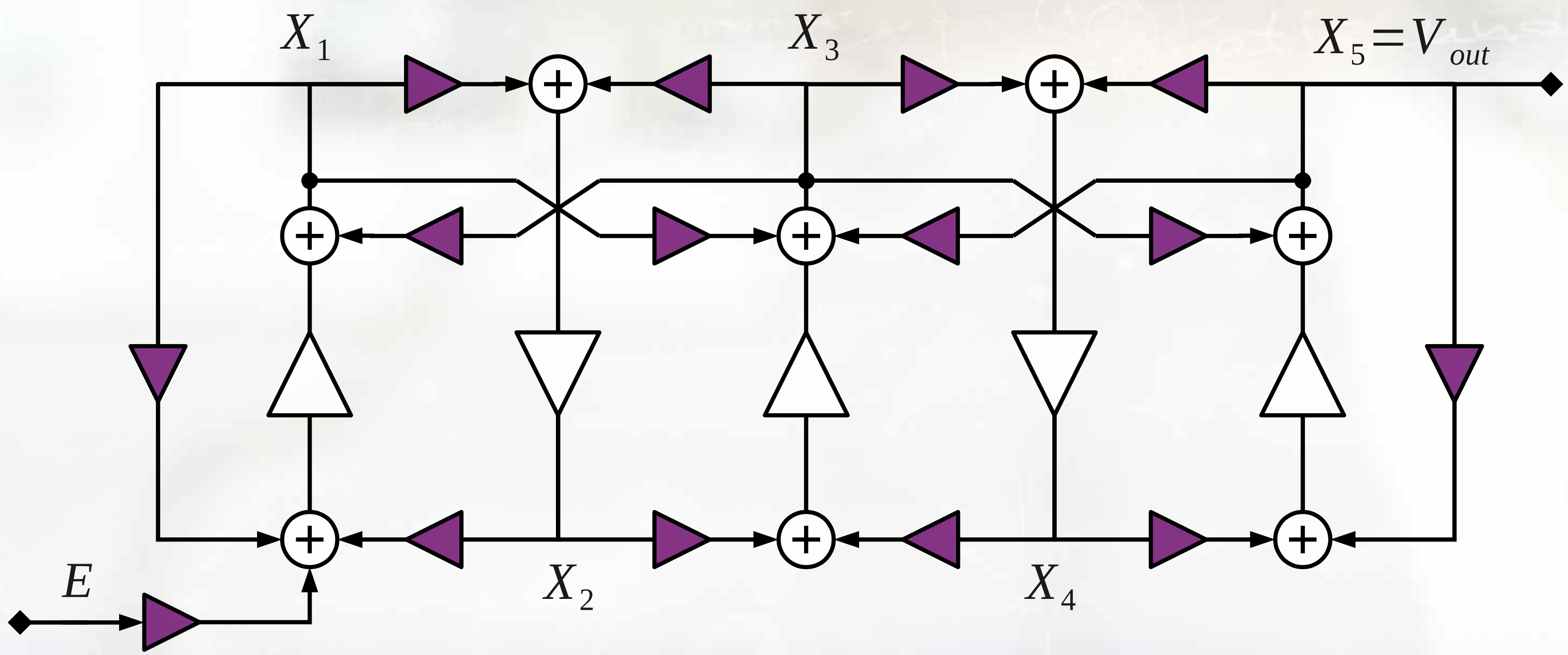




Optimum wrt. sensitivity (exercises in lessons)

State-space representation

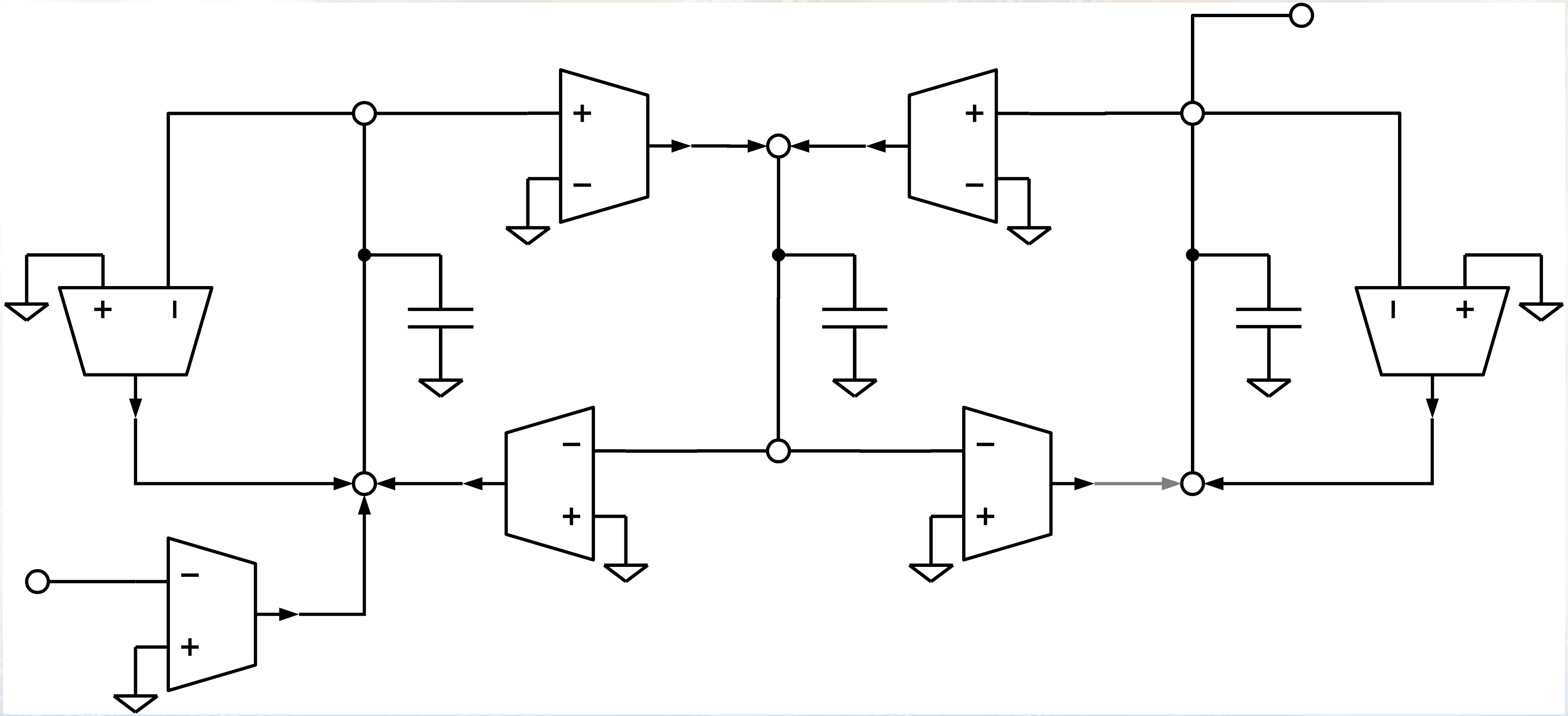
Note the voltages and currents through the ladder network



$$E = V_{in}, X_1 = V_1, X_2 = R \cdot I_2, X_3 = V_3, X_4 = R \cdot I_4, \ldots$$

Continuous-time filter implementation

Gm-C



Leapfrog filters

See the handouts for state-space realization and implementations

http://www.es.isy.liu.se/courses/ANIK/download/filterRef/ANTIK_0NNN_LN_leapfrogFiltersOH1_A.pdf

http://www.es.isy.liu.se/courses/ANIK/download/filterRef/ANTIK_0NNN_LN_leapfrogSynthesisExtra1_A.pdf

http://www.es.isy.liu.se/courses/ANIK/download/filterRef/ANTIK_0NNN_LN_leapfrogSynthesisExtra2_A.pdf

http://www.es.isy.liu.se/courses/ANIK/download/switcapRef/ANTIK_0NNN_LN_switcapHandout_B.pdf

What did we do today?

Switched capacitor circuits with nonideal effects in mind

- What should we look out for?
- What is the impact on system performance, like filters.

Continuous-time filters

- The way forward, and the background to generate the filters.
- OTA-C, Gm-C, and active-RC

What will we do next time?

Continuous-time filters

- Wrap-up and some more conclusions

Discrete-time filters

- Simulation of the continuous-time filters