Lecture 6, ANIK

Operational amplifiers
Stability, compensation, etc.

LiU EXPANDING REALITY

What did we do last time?

Differential signals

- Why differential?
- Common-mode definitions

Differential pair

- Analysis
- Operation

Mismatch

- Impact of mismatch on design/performance/behavior

What will we do today?

Operational amplifiers

- Stability
- Frequency analysis
- Compensation

Operational amplifiers

Operational amplifiers

- drive resistive loads
- have zero output impedance
- act like a voltage source

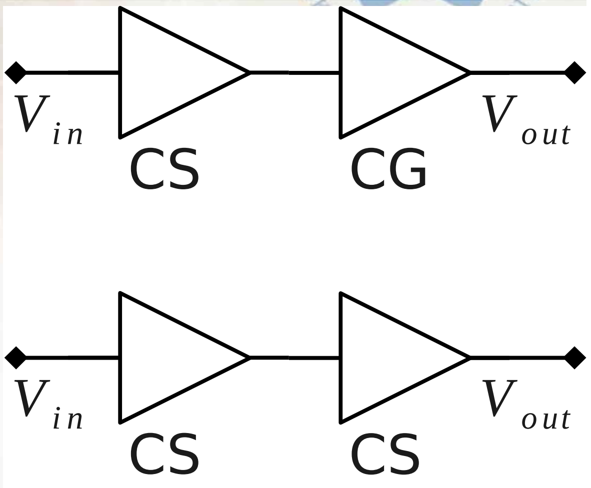
Operational transconductance amplifiers

- drive capacitive loads
- have infinite output impedance
- act like a current source

Gain increased with multi-stage amplifiers

Single-stage (cascodes) vs two-stage?

- They will have the same DC gain
- They will not have the same output impedance
- Multiple poles ("one per stage")



The transfer function (in both cases) is

$$A(s) = \frac{A_1 \cdot A_2}{\left(1 + \frac{s}{p_{11}}\right) \cdot \left(1 + \frac{s}{p_{12}}\right)}$$

Multiple poles

Case 1 (CS+CG)

First amplifier sees low-impedance load: $(g_1 + g_{m1}) \| C_1 \approx g_{m1} \| C_1$

Second amplifier sees capacitive load: $g_{out} \approx g_2 \| C_2$

Case 2 (CS+CS)

First amplifier sees high-impedance load $(g_1 + 0) \| C_1 \approx g_1 \| C_1$

Second amplifier sees capacitive load: $g_{out} \approx g_2 \| C_2$

Notice that the g_2 in Case 2 is higher than g_2 in Case 1.

Regardless what you do ... Feedback

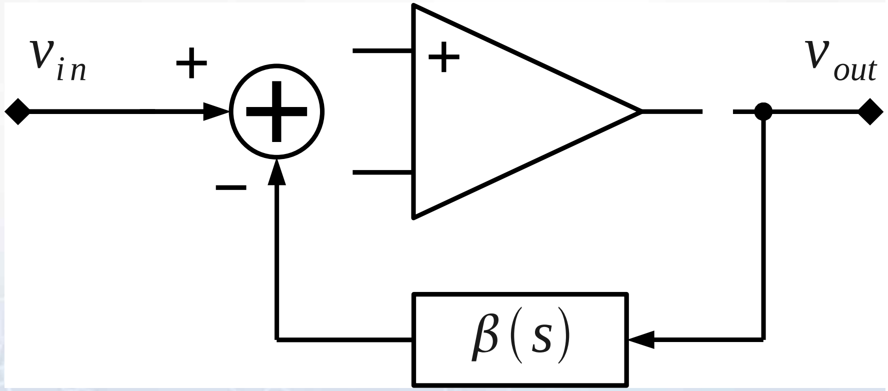
Preferrably, we have a controlled system with a closed-loop gain of:

$$Y(s) = \left(X(s) - \beta(s) \cdot Y(s)\right) \cdot A(s) \Rightarrow$$

$$\frac{Y(s)}{X(s)} = \frac{A(s)}{1 + \beta(s) \cdot A(s)} = \frac{1/\beta(s)}{1 + \frac{1}{\beta(s) \cdot A(s)}}$$

A feedback factor of: $\beta(s)$

An open-loop gain of: $\beta(s) \cdot A(s)$

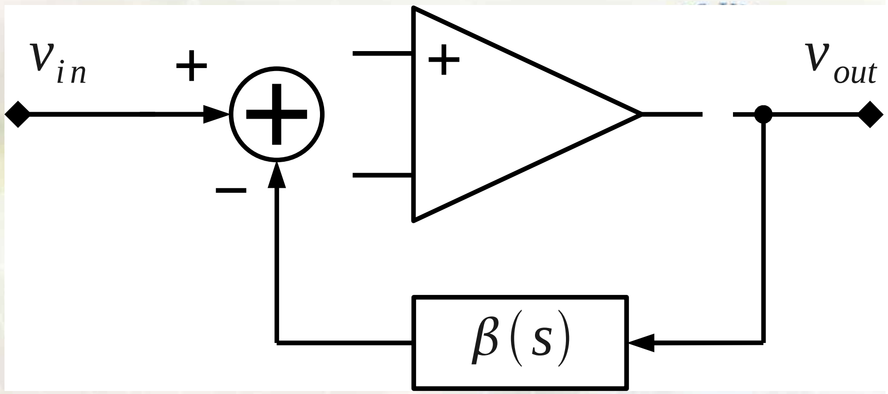


LiU EXPANDING REALITY

Why do you want controlled feedback?

Gain is now under control!

- No variation with gm / gds, instead given by (normally) high-accuracy components



"Unlimited" drive capability

Isolation of input and output

Linearization

Remember, it is a regulation loop. It is designed to track the changes, anything added in the loop will be supressed.

The problem: Stability

In short: the transfer function must be designed such that

$$\beta(s)\cdot A(s)\neq -1$$

If this is the case, we have an infinitely high transfer function

(In reality, the proof is quite complex.)

Phase margin (how far are we off from this to happen)

Poor phase margin gives ringing in the output when applying step

Critically damped signal at approximately 70 degrees (poles become real rather than complex pair, i.e., they are well splitted)

Stability, cont'd

Bode plot

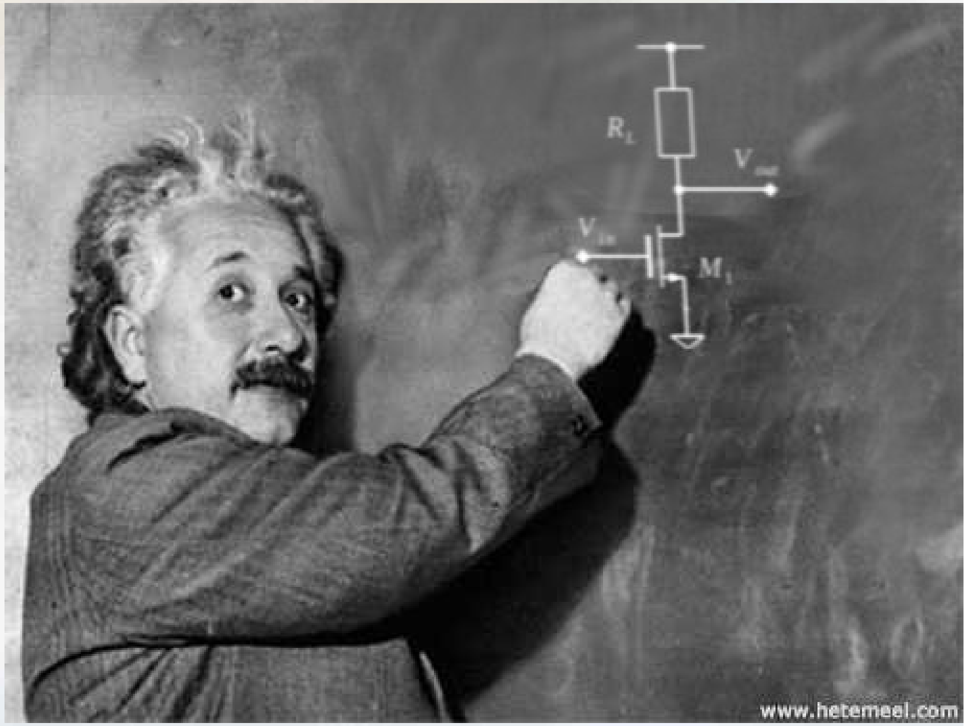
- What happens to the transfer characteristics?

- Phase margin

Step response

- Settling
- Oscillations

Critically damped at 70 degrees

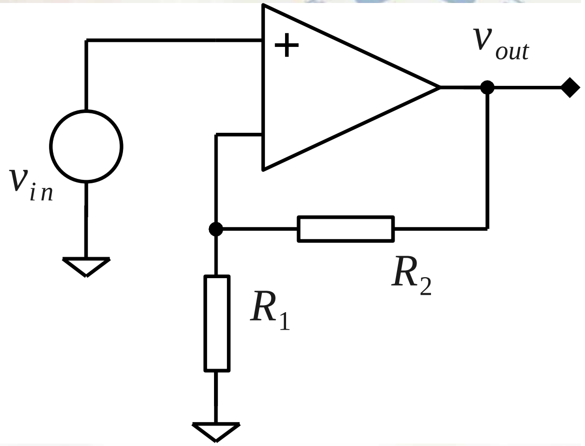


Settling time vs phase margin

What's behind the story?

Ideal case:

$$\frac{R_1}{R_1+R_2}\cdot v_{out}=v_{in}\Rightarrow\frac{v_{out}}{v_{in}}=\frac{R_1+R_2}{R_1}=\Gamma$$



Non-ideal gain case:

$$v_{out}=A_0\cdot\left[v_{in}-\frac{R_1}{R_1+R_2}\cdot v_{out}\right]\Rightarrow\frac{v_{out}}{v_{in}}=\frac{1}{\frac{1}{A_0}+\frac{R_1}{R_1+R_2}}=$$

$$=\frac{R_1+R_2}{R_1}\cdot\frac{1}{1+\frac{R_1+R_2}{A_0\cdot R_1}}=\frac{\Gamma}{1+\frac{\Gamma}{A_0}}$$

What's behind the story, cont'd?

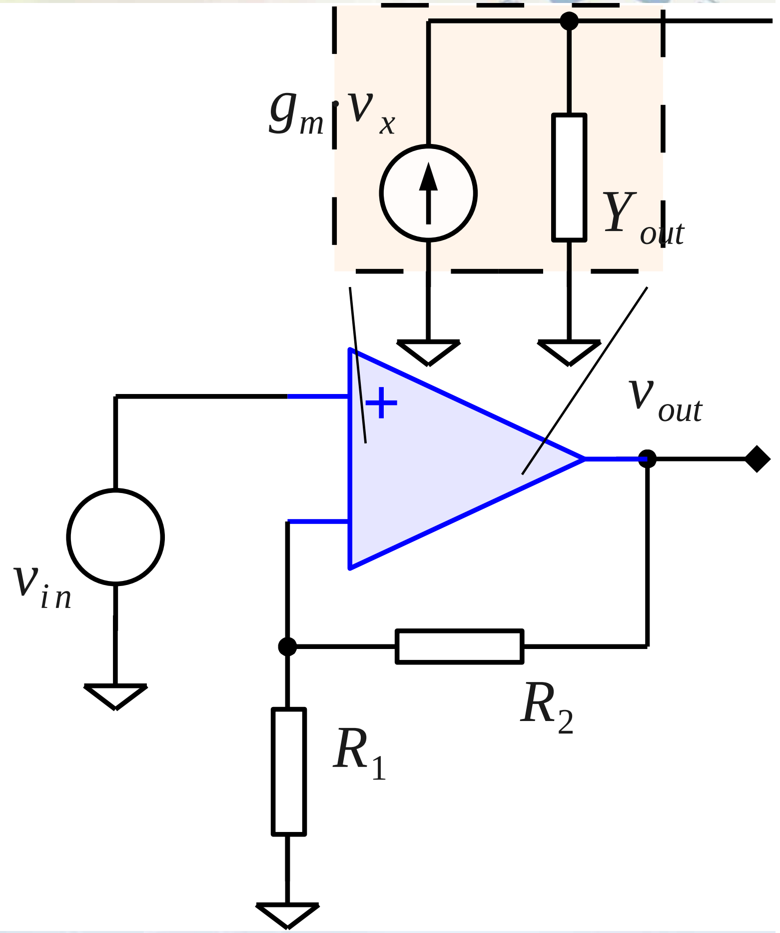
Single-pole model (ignore the effect of output impedance mismatch):

$$\frac{v_{out}}{v_{in}} = \frac{\Gamma}{1 + \frac{\Gamma}{A_0} \cdot \left(1 + \frac{p_1}{s}\right)}$$

Results in

$$\frac{v_{out}}{v_{in}} = \frac{\frac{\Gamma}{1 + \Gamma / A_0}}{1 + \frac{\Gamma / A_0}{1 + \Gamma / A_0} \cdot \frac{p_1}{s}} \approx \frac{\Gamma}{1 + \frac{p_1}{s}}$$

The amplifier will band-limit the system!



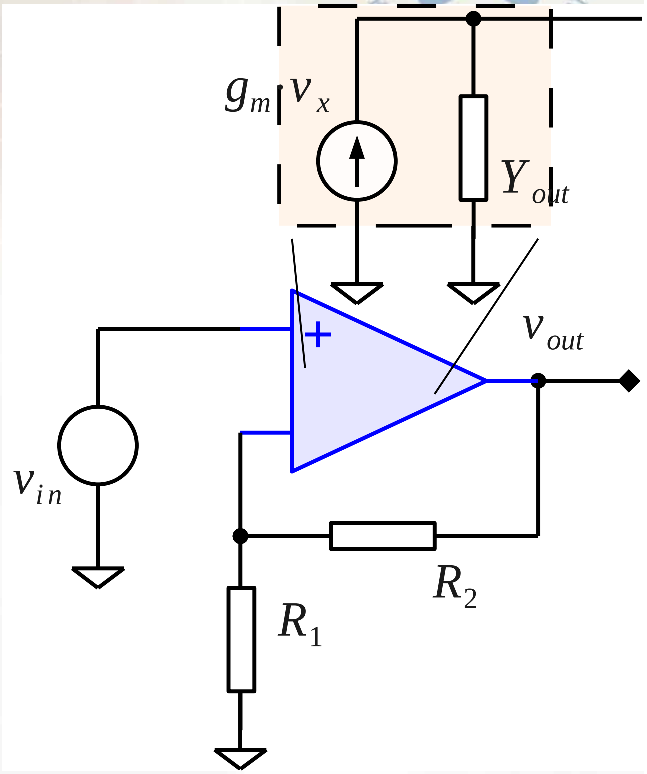
What's behind the story, cont'd?

Any-pole model:

$$v_x = \frac{R_1}{R_1 + R_2} \cdot v_{out}$$

$$g_m (v_{in} - v_x) + (0 - v_{out}) Y_{out} + \frac{0 - v_{out}}{R_1 + R_2} = 0$$

$$g_m v_{in} = v_{out} \cdot \left[Y_{out} + \frac{1 + R_1 g_m}{R_1 + R_2} \right]$$

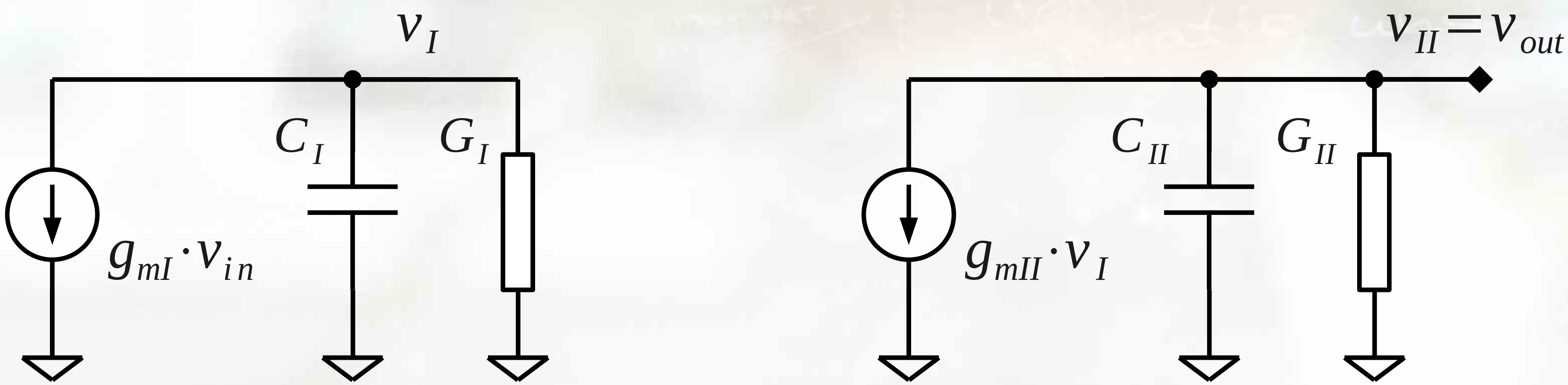


Which boils down to something more complex:

$$\frac{v_{out}}{v_{in}} = \frac{\Gamma}{1 + \frac{1}{g_m R_1} + \frac{\Gamma}{g_m / Y_{out}}}$$

We need to be a bit more systematic

The model (high-impedance load) and focus on two-pole systems



$$p_1 = \frac{G_I}{C_I}, \quad p_2 = \frac{G_{II}}{C_{II}}, \quad A_1 = \frac{g_{mI}}{G_I}, \quad A_2 = \frac{g_{mII}}{G_{II}}$$

Dominant pole assumption (output)

Assuming pole splitting, $p_2 \gg p_1$, gives us

$$A(s) = \frac{A_1 \cdot A_2}{\left(1 + \frac{s}{p_{11}}\right) \cdot \left(1 + \frac{s}{p_{12}}\right)} \approx \frac{A_1 \cdot A_2}{1 + \frac{s}{p_1} + \frac{s^2}{p_1 \cdot p_2}}$$

This implies: $\omega_{ug} \approx A_1 \cdot A_2 \cdot p_1$ and

$$\phi_m = 180 - arg\, A(j\omega_{ug}) = 180 - atan\frac{\omega_{ug}}{p_1} - atan\frac{\omega_{ug}}{p_2} \approx 90 - atan\frac{\omega_{ug}}{p_2}$$

$$\phi_m \approx 90 - atan\left(A_0 \cdot \frac{p_1}{p_2}\right)$$

The formulas (dominant load!)

Unity-gain frequency

$$\omega_{ug} \approx \frac{g_{mI} \cdot g_{mII}}{G_I \cdot G_{II}} \cdot \frac{G_{II}}{C_{II}} = \frac{g_{mI} \cdot g_{mII}}{G_I \cdot C_{II}}$$

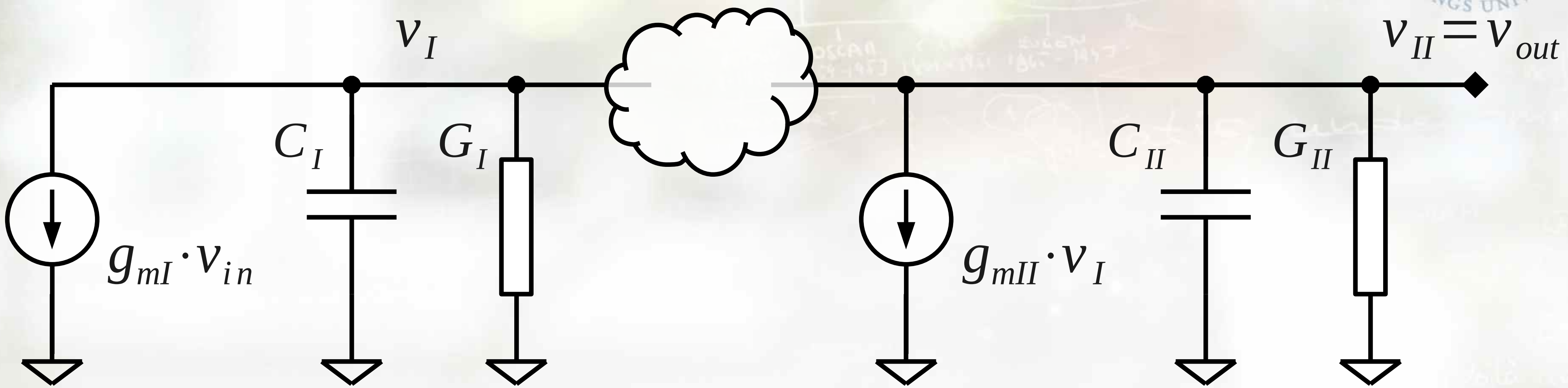
Phase margin

$$\phi_m \approx 90 - atan \frac{\omega_{ug}}{p_2} = 90 - atan \frac{\dfrac{g_{mI} \cdot g_{mII}}{G_I \cdot C_{II}}}{\dfrac{G_I}{C_I}} = 90 - atan \frac{g_{mI} \cdot g_{mII} \cdot C_I}{G_I^2 \cdot C_{II}}$$

etc., etc., etc.

We need to be a bit more organized...

Compensation, poles are too close



The "cloud" is typically a capacitor or a series resistor-capacitor.

Compensation, Miller capacitance

Introduced zero	Parasitic pole	Dominant pole	Unity-gain
$z_1 = \frac{g_{mII}}{C_C}$	$p_2 = \frac{-g_{mII}}{C_{II}}$	$p_1 = \frac{-G_I \cdot G_{II}}{g_{mII} \cdot C_C}$	$\omega_{ug} = \frac{g_{mI}}{C_C}$

Introduced zero	Parasitic pole	Phase margin
$z_1 \approx 10 \cdot \omega_{ug}$	$p_2 \approx 2.2 \cdot \omega_{ug}$	≈ 60

Dominant pole moves "down", parasitic pole moves "up"

Parasitic zero added (harmful for phase margin)

Compensation, Nulling resistor 1

Introduced zero	Parasitic poles	Dominant pole	Unity-gain
$z_1 = \frac{g_{mII}}{C_C} \cdot \frac{1}{1 - R_Z \cdot g_{mII}}$	$p_2 = \frac{-g_{mII}}{C_{II}}$, $p_3 = \frac{-1}{R_Z \cdot C_{II}}$	$p_1 = \frac{-G_I \cdot G_{II}}{g_{mII} \cdot C_C}$	$\omega_{ug} = \frac{g_{mI}}{C_C}$

$$R_Z = \frac{1}{g_{mII}} \cdot \left(1 + \frac{C_{II}}{C_C}\right)$$

Introduced zero	Parasitic pole	Phase margin
$z_1 \rightarrow p_2$	$p_3 \approx 1.73 \cdot \omega_{ug}$	≈ 60

Compensation, Nulling resistor 2

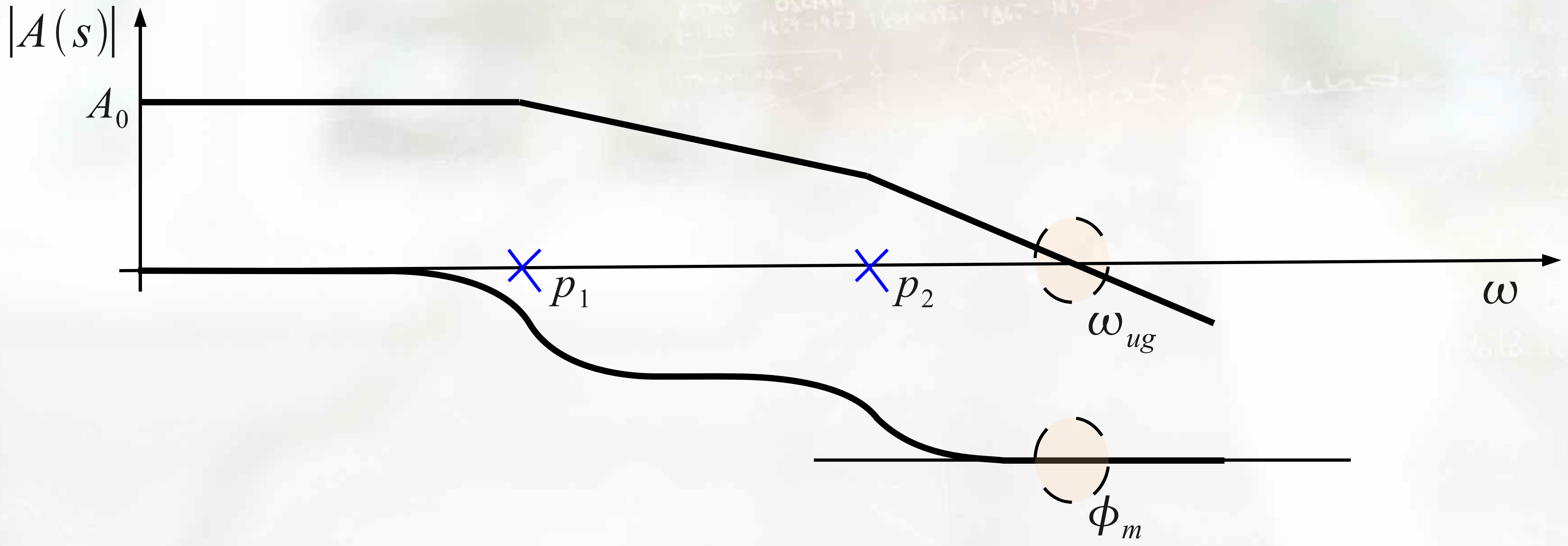
Introduced zero	Parasitic poles	Dominant pole	Unity-gain
$z_1 = \frac{g_{mII}}{C_C} \cdot \frac{1}{1 - R_Z \cdot g_{mII}}$	$p_2 = \frac{-g_{mII}}{C_{II}}$, $p_3 = \frac{-1}{R_Z \cdot C_{II}}$	$p_1 = \frac{-G_I \cdot G_{II}}{g_{mII} \cdot C_C}$	$\omega_{ug} = \frac{g_{mI}}{C_C}$

$$R_Z = \frac{1}{g_{mII}}$$

Introduced zero	Parasitic pole	Phase margin
$z_1 \rightarrow \infty$	$p_2 \approx 1.73 \cdot \omega_{ug}$, $p_3 > 10 \cdot \omega_{ug}$	≈ 60

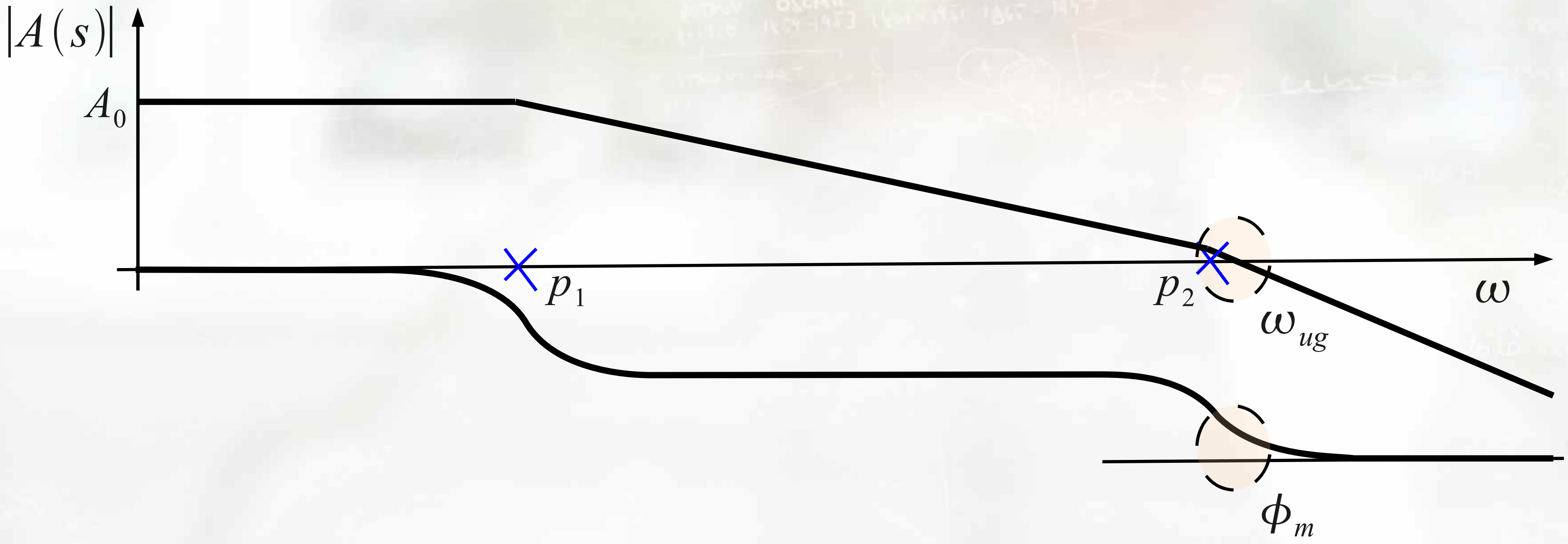
Poles and zeros revisited

Stable?



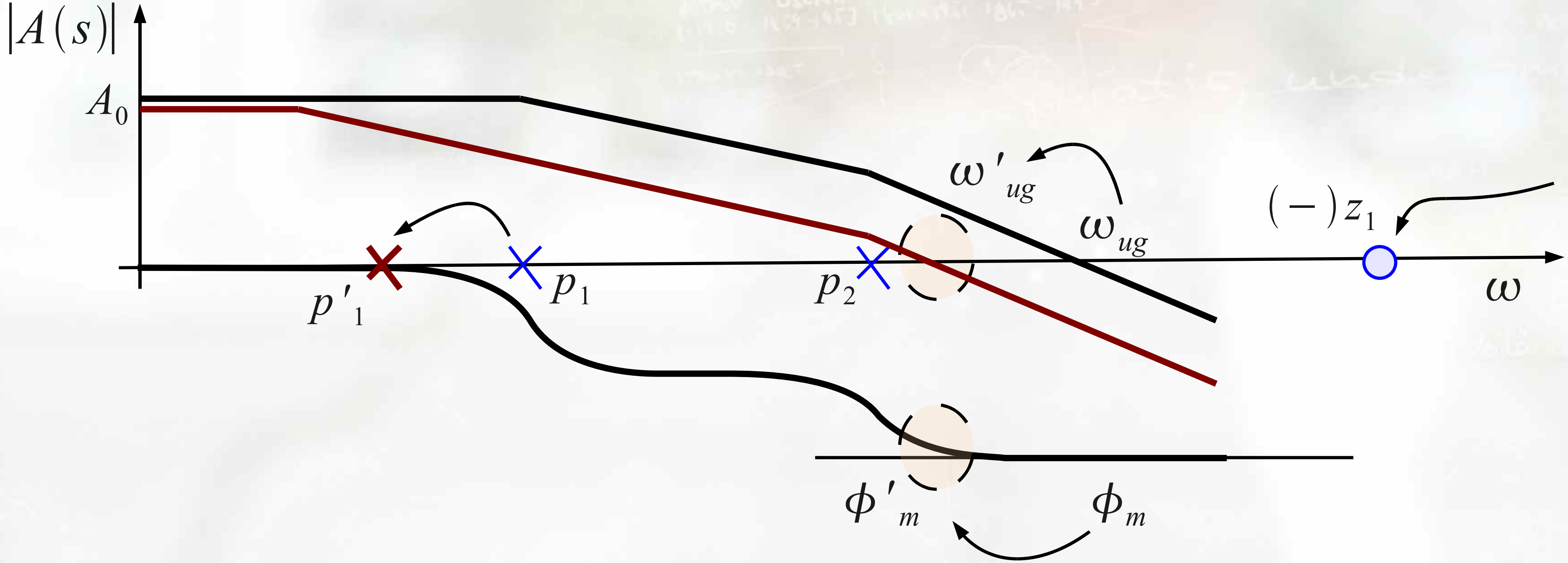
Poles and zeros revisited

Stable?



Compensation

What is the cost associated with compensation?



Compensation, two cases:

1) "Internal" node sees a low-impedance node

Typically: output load dominates, and we should drive a capacitive load

Load-compensation, i.e., increase cap externally

2) "Internal" node sees a high-impedance node

Typically: internal load dominates, and we should drive a resistive load

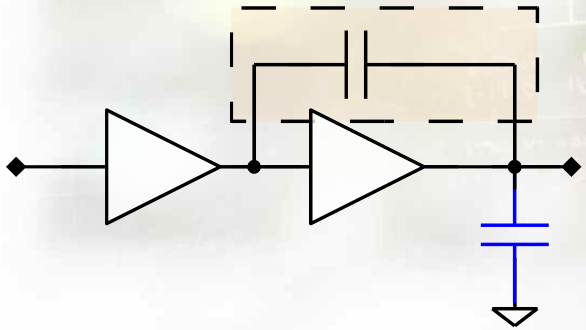
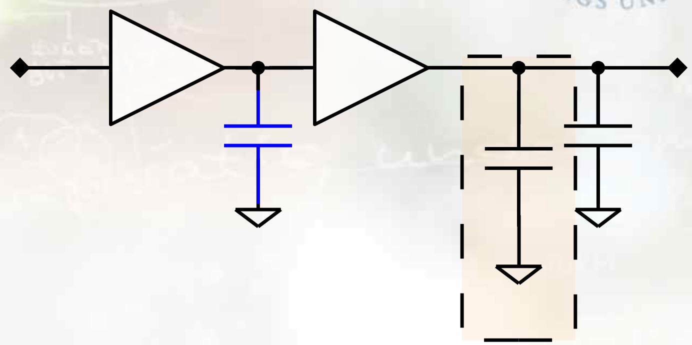
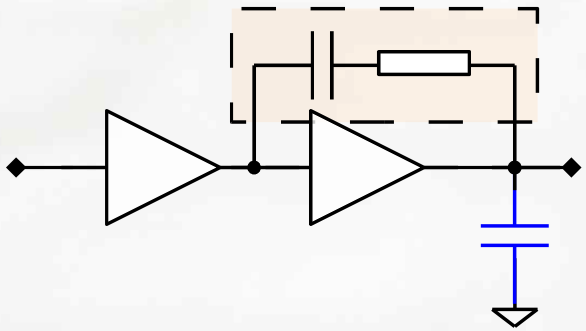
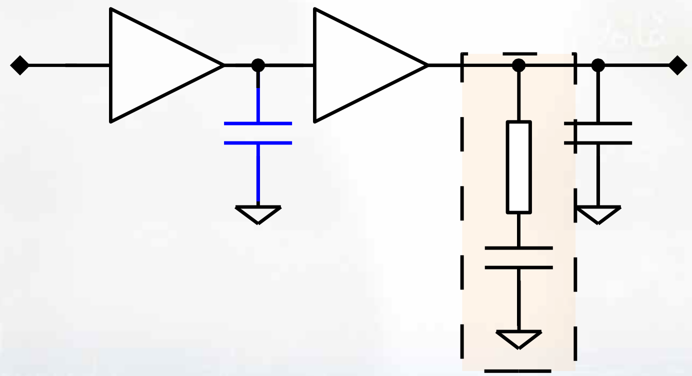
Miller-compensation, i.e., utilize the second-stage gain to multiply C_C

As always, some exceptions to the rule:

We could add common-drain at output

Nested compensation, active compensation, ... and more ...

Compensation compiled:

	Miller	Load compensation
Cap		
Cap + Res		

Rule-of-thumbs for hand-calculation

Use MATLAB or similar to support your calculations for better understanding

- See for example
  `/site/edu/es/ANTIK/antikPoleZero.m`
  `/site/edu/es/ANTIK/antikSettling.m`

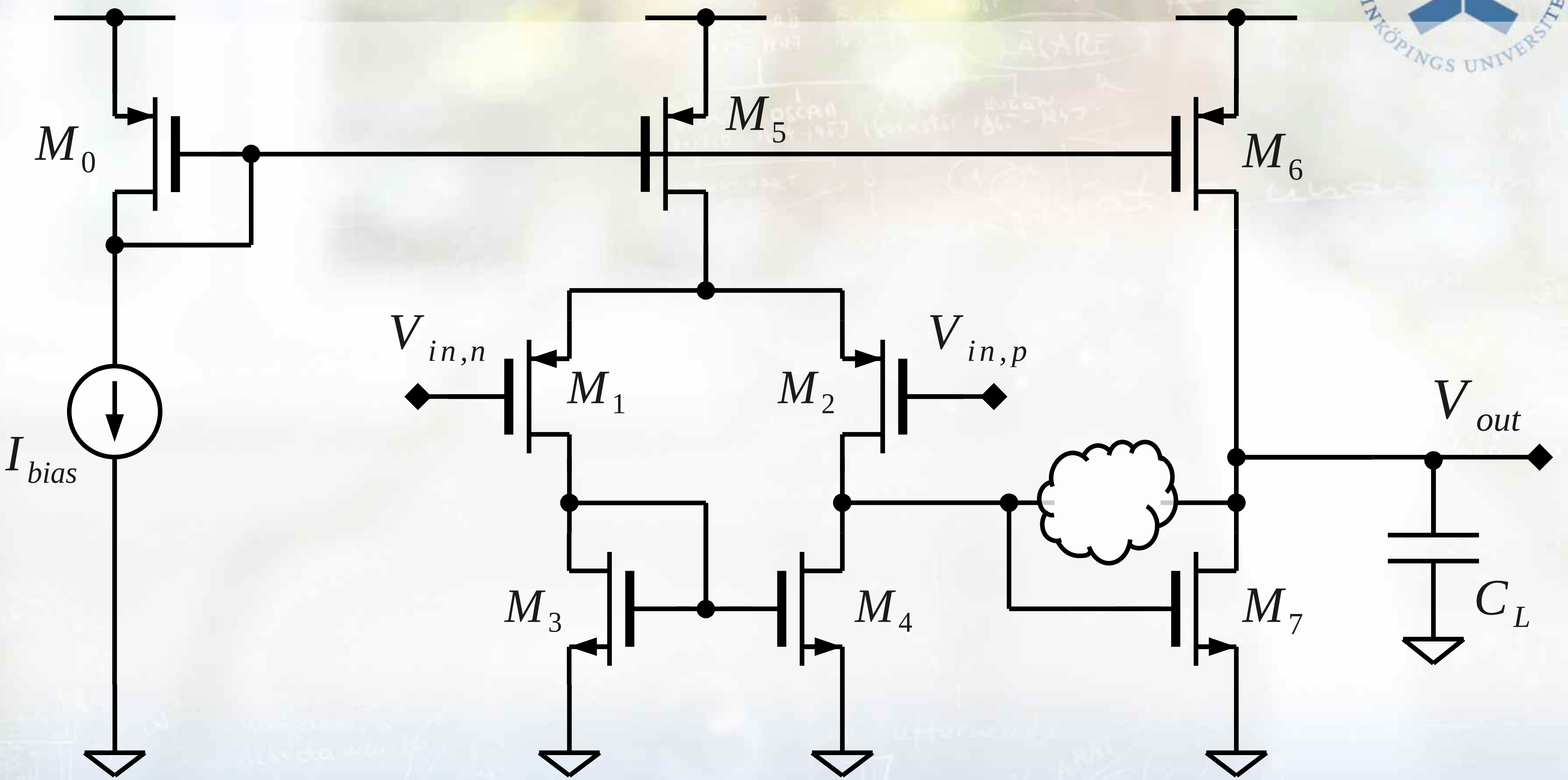
In the end, use the simulator.

- It has to be robust over several corners, temperatures, and other variations.
- Hand calculations are incorrect per definition

Model corresponds quite well with circuit once you have identified the different stages

- See for example exercises

Operational amplifer



What did we do today?

Operational amplifiers

- Top-level aspects
- Compensation
- Stability

What will we do next time?

Operational amplifiers

- Circuit-level aspects