Lesson 7

Lesson Exercises: **B15.1-3, B15.10 - B15.15, K11, K12, K16, K22**

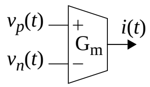
Recommended Exercises: *K10, K17, K18, K19, K20, K21*

Theoretical Issues: **Filtersyntes med G_m-C element.**

Theoretical

• Integrators. G_m-C building blocks

G_m-C Element

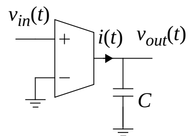


Current flowing out of the Gm-C element id equal to the transconductance times the voltage over the input.

$$i(t) = G_m \cdot [v_p(t) - v_n(t)] = G_m \cdot v_i(t)$$

Ideally, the input current is zero, hence an infinite input impedance. The output impedance is considered to be zero.

Integrator

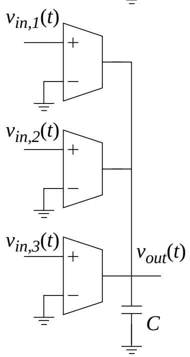


For an integrator we use a grounded capacitance. Assuming that the output is connected to a high impedance node, the output current from the Gm-C is described by the two relations

$$i(t) = C\frac{dv_{out}(t)}{dt}$$

and

$$i(t) = G_m \cdot v_{in}(t)$$



The output voltage is then simply found by substituing and integrate

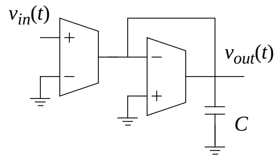
$$v_{out}(t) = \frac{G_m}{C}\int v_{in}(t)dt$$

The summing integrator's output is found in the same way

$$v_{out}(t) = \frac{1}{C}\sum_{k=1}^{N} G_{m,k}\int v_{in,k}(t)dt$$

The $G_{m,k}/C$ factor is the scaling factor for the input voltage k.

A first order section



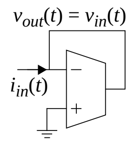
Assume that we want to realize a single pole system according to

$$H(s) = \frac{a_0}{s + b_0}$$

The transfer function of the G_m-C circuit is

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{g_{m1}/C}{s + g_{m2}/C}$$

In fact the G_m-C with feedback operates as a grounded resistor. Consider:



$$v_{in}(t) = v_{out}(t)$$

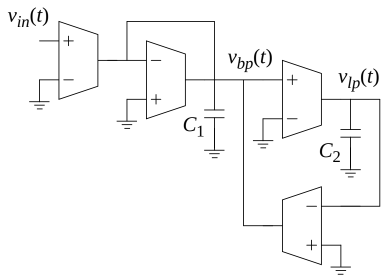
$$i_{out}(t) = -g_m \cdot v_{in}(t)$$

$$i_{in}(t) = -i_{out}(t)$$

Which gives

$$\frac{v_{in}(t)}{i_{in}(t)} = \frac{1}{g_m} \text{ with } R \equiv \frac{1}{g_m}$$

A second order section



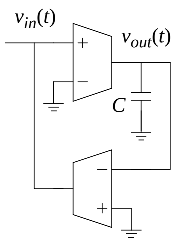
Assume that we want to realize a single pole system according to

$$H(s) = \frac{a_1 s + a_0}{s^2 + b_1 s + b_0}$$

The solution to this problem is shown in the figure. We can use two voltages directly from the circuit, the v_{bp} and v_{lp}. They implement a band pass and low pass filtering.

$$\frac{V_{bp}(s)}{V_{in}(s)} = \frac{s\frac{g_{m1}}{C_1}}{s^2 + s\frac{g_{m2}}{C_1} + \frac{g_{m3}g_{m4}}{C_1C_2}} \text{ and } \frac{V_{lp}(s)}{V_{in}(s)} = \frac{\frac{g_{m1}g_{m3}}{C_1C_2}}{s^2 + s\frac{g_{m2}}{C_1} + \frac{g_{m3}g_{m4}}{C_1C_2}}$$

The two G_m-C circuits in the feedback structure function as a grounded inductor.



$$i_{in}(t) = -i_2(t)$$

$$i_2(t) = -g_{m2} \cdot v_{out}(t)$$

$$v_{out}(t) = \frac{1}{C}\int i_1(t)dt = \frac{g_{m1}}{C}\int v_{in}(t)dt$$

Combining the equations gives

$$\frac{i_{in}(t)}{g_{m2}} = \frac{g_{m1}}{C}\int v_{in}(t)dt$$

or

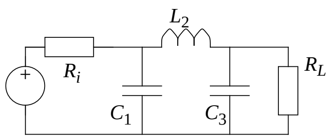
$$v_{in}(t) = \frac{C}{g_{m1} \cdot g_{m2}} \cdot \frac{di_{in}(t)}{dt} \text{ with } L \equiv \frac{C}{g_{m1} \cdot g_{m2}}$$

• Leapfrog filters (Gyrator filters)

Ladder Filters

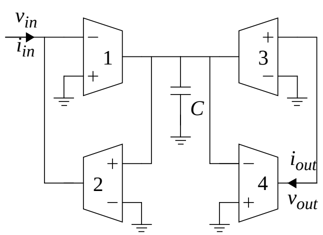
The ladder filter structure with capacitances and inductors. We let the source and load have resistances.

We also now have a floating inductor, that has to be considered.



Floating inductors

The floating inductor can be realized by using the following structure:



$$I_{in} = -I_2 = -G_{m2}V_C$$

$$I_{out} = -I_4 = -(-G_{m4}V_C) = G_{m4}V_C$$

$$V_C = \frac{I_C}{sC}$$

$$I_C = I_1 + I_3 = -G_{m1}V_{in} + G_{m3}V_{out}$$

For the inductor, naturally, $I_{in} = -I_{out}$. This forces $G_{m2} = G_{m4} = G_{m24}$.

$$V_{out} = V_{in} \cdot \frac{G_{m1}}{G_{m3}} + sC \cdot V_C = V_{in} \cdot \frac{G_{m1}}{G_{m3}} - sC \cdot \frac{I_{in}}{G_{m24}}$$

Compare this with a true inductor

$$V_{out} = V_{in} - sL \cdot I_{in}$$

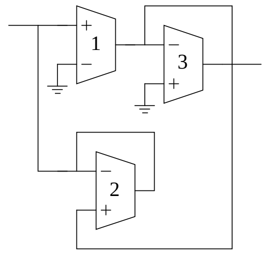
By identifying the terms we see that

$$G_{m1} = G_{m3} = G_{m13}$$

and that the simulated inductor value is

$$L \equiv \frac{C}{G_{m24}}$$

We see however that we also have to implement a floating resistor with G_m-C elements. Compare with the first order section as well. This is done by using the following structure.



$$I_{in} = -I_2$$

$$I_2 = G_{m2} \cdot (V_{out} - V_{in})$$

$$I_{in} = G_{m2} \cdot (V_{in} - V_{out})$$

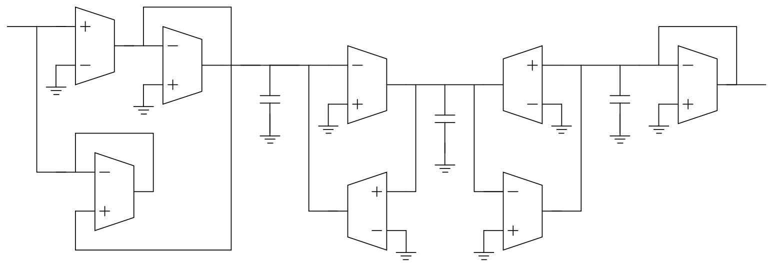
We directly see that

$$R \equiv \frac{1}{G_{m2}}$$

We have to guarantee that $I_{out} = -I_{in}$ as well.

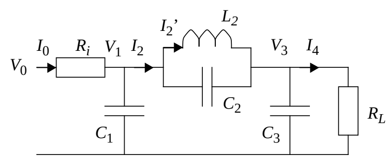
$$-I_{out} = I_1 + I_3 = G_{m1} \cdot V_{in} - G_{m3} \cdot V_{out}$$

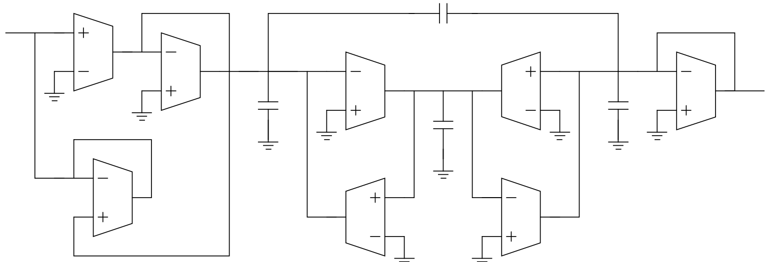
The currents are equal when $I_{out} = -I_{in}$ when $G_{m1} = G_{m3} = G_{m2}$



Elliptic filters

For the elliptic filter we have a slightly different situation. We have a floating inductor in parallel with a floating capacitance. A simple, but naive, implementation is to simply use a capacitance.

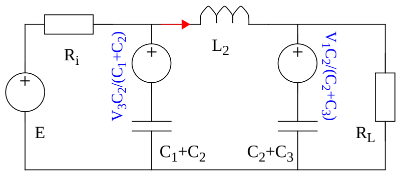




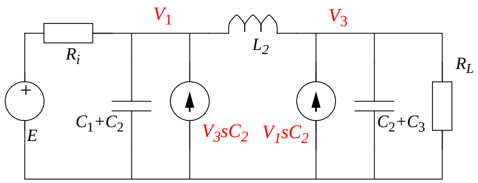
State-Variable Filters

For on-chip implementations it is however more suitable to use grounded capacitances (less parasitic capacitances).

We do the same transformation as in the previous lesson. A pair of voltage sources is introduced in the net.



Using Norton equivalents, this is transformed into

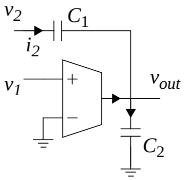


The inductor is still implemented with a gyrator.

The capacitances, $C_1 + C_2$ and $C_2 + C_3$ are still grounded. We have to realize the current sources. This can be done by using operational amplifiers together with the G_mC element.

Exercises

Exercise K10



The equations for the circuit are given by

$$I_2 = sC_1 \cdot (V_2 - V_{out})$$

$$I_x = G_m \cdot V_1$$

$$I_2 + I_x = sC_2 \cdot V_{out}$$

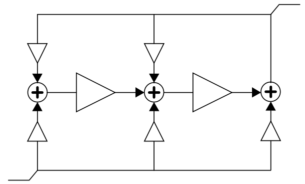
$$V_{out} = \frac{1}{sC_2} \cdot (I_2 + I_x) = \frac{C_1}{C_2}(V_2 - V_{out}) + \frac{G_m}{sC_2} \cdot V_1$$

$$V_{out} = \frac{G_m}{s\alpha}V_1 + \frac{C_1}{\alpha}V_2 \text{ where } \alpha = C_1 + C_2$$

With this circuit we can perform an addition and an integration.

Exercise K11

Realize the filter having the transfer function

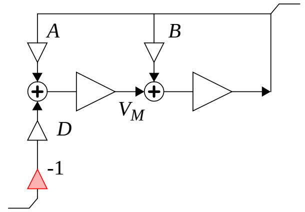


$$H(s) = \frac{1\times10^6}{s^2 + 3\times10^5 \cdot s + 6\times10^6}$$

We rewrite the function as

$$Y(s) \cdot [s^2 + 3\times10^5 \cdot s + 6\times10^6] = 1\times10^6 \cdot X(s)$$

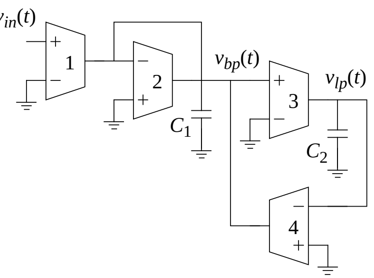
Or



$$Y(s) = -\frac{3\times10^5}{s}Y(s) - \frac{6\times10^6}{s^2}Y(s) + \frac{1\times10^6}{s^2}X(s)$$

The flow graph is transformed. $A = -6\times10^6$, $B = -3\times10^5$ and $D = -1\times10^6$. Note the insertion of the inverter. This can now be used to implement the active filter.

A second order section can be used. From the theory we know that



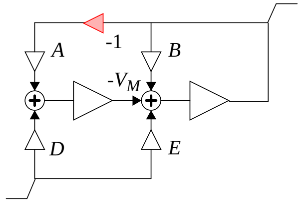
$$\frac{V_{lp}(s)}{V_{in}(s)} = \frac{\frac{g_{m1}g_{m3}}{C_1C_2}}{s^2 + s\frac{g_{m2}}{C_1} + \frac{g_{m3}g_{m4}}{C_1C_2}}$$

We can directly identify the values from by comparing the equations. Choose for example all capacitors equal, e.g. $C = 1\times10^{-9}$ F

Exercise K12

Realize the filter having the transfer function

$$H(s) = \frac{-1\times10^6(s-1)}{s^2 + 3\times10^5 \cdot s + 6\times10^6}$$



The function is rewritten in the same manner as in the previous exercise. In this case we however have a slightly different structure.

$$Y(s) = -\frac{3\times10^5}{s}Y(s) - \frac{6\times10^6}{s^2}Y(s) - \frac{1\times10^6}{s}X(s) + \frac{1\times10^6}{s^2}X(s)$$

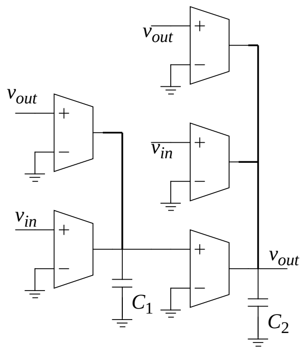
We assume that we feed back the positive output (constant A) and we construct with a negative intermediate node, $-V_M$.

With

$A = 6\times10^6$, $B = -3\times10^5$, $D = 1\times10^6$ and

$E = -1\times10^6$

In this implementation we will follow the signal flow graph



Exercise K16

Synthesize an active elliptic leapfrog filter. Termination resistances are $1k\Omega$. Specification gives:

Pass band: $0 < \omega < 2\pi$ krad/s, $A_{max} = 0.1$ dB

Stop band: $\omega > 4\pi$ krad/s, $A_{min} > 20$ dB

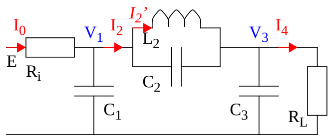
Order is found with table to be $N = 3$.

This gives following filter structure. Component values are found to be

$C_1' = C_3' = 0.8740$, $C_2' = 0.2411$ och

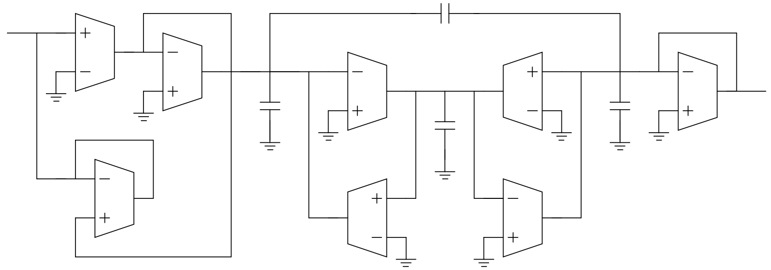
$L_2' = 0.9083$, and $R_i = R_L = 1k\Omega$,

$\kappa^2 = 1$.



Denormalized values are given by

$$C = \frac{C_n}{\omega_0 R_0} \text{ and } L = \frac{R_0}{\omega_0}L_n \text{ give } C_1 = C_3 \approx 139.1nF,\ C_2 \approx 38.4nF,\ L_2 \approx 144.6mH$$



Exercise B15.10

Extended to the exercise: $(W/L)_1 = (W/L)_2 = 5$.

Note that there is a constant current, I_1, flowing through Q_1 and Q_2. Assume that they are operating in their saturation region, that they have the same transconductance parameter, $\beta_{12} = \beta_1 = \beta_2$, and that they have the same threshold voltage, $v_{T,12}$, hence

$$I_1 = \frac{\beta_1}{2}\cdot(v_{GS,1} - v_{T,12})^2 = \frac{\beta_2}{2}\cdot(v_{GS,2} - v_{T,12})^2$$

$$v_{GS,1} = v_{GS,2} = \sqrt{\frac{2I_1}{\beta_{12}}} + v_{T,12}$$

The source voltage at Q_1 may naturally be written as $v_{S,1} = v_i^+ - v_{GS,1}$ and correspondingly for the source voltage at Q_2. Therefore, the voltage across Q_9 must be

$$(v_i^+ - v_{GS,1}) - (v_i^- - v_{GS,2}) = v_i^+ - v_i^- = -v_{DS,9}$$

The current through Q_9 must be given by (the transistor is working in its triode region)

$$i_{D,9} = \frac{\beta_9}{2}\cdot(2(v_{GS,9} - v_{T,9}) - v_{DS,9})\cdot v_{DS,9}$$

This is approximately

$$i_{D,9} \approx \beta_9\cdot(v_{GS,9} - v_{T,9})\cdot v_{DS,9} = -\beta_9\cdot(v_{GS,9} - v_{T,9})\cdot(v_i^+ - v_i^-)$$

The conductance is given by

$$g_{ds} = \frac{i_{D,9}}{v_i^+ - v_i^-} = \beta_9\cdot(v_{GS,9} - v_{T,9})$$

We see that the current through Q_4 must be $I_1 + i_{D,9}$ and the current is mirrored to the output and therefore, $I_1 + i_{D,9}$ must flow through Q_8 as well. This indicates that $i_{o1} = i_{D,9}$ and we see from the equations that the G_m of the total circuit is given by

$$G_m = g_{ds}$$

It may be rewritten

$$i_{D,9} = -\beta_9 \cdot (V_C - v_{S,9} - v_{T,9}) \cdot (v_i^+ - v_i^-) = -\beta_9 \cdot (V_C - v_{S,1} - v_{T,9}) \cdot (v_i^+ - v_i^-)$$

or

$$G_m = g_{ds} \approx \beta_9 \cdot (V_C - v_{S,1} - v_{T,9})$$

The threshold voltage $v_{T,9}$ is given by

$$v_{T,9} = V_{T0} + \gamma \cdot (\sqrt{2|\phi_F| + v_{SB,9}} - \sqrt{2|\phi_F|}) =$$
$$= V_{T0} + \gamma \cdot (\sqrt{2|\phi_F| + v_{S,1}} - \sqrt{2|\phi_F|}) = v_{T,1}$$

This gives that we can rewrite as

$$G_m = \beta_9 \cdot (V_C - (v_i^+ - v_{GS,1}) - v_{T,9}) = \beta_9 \cdot ((V_C - v_i^+) + (v_{GS,1} - v_{T,1})) =$$
$$= \beta_9 \cdot [(V_C - v_i^+) + \sqrt{2I_1/\beta_1}]$$

Values taken from page 78 in the text book give

$$G_m = (92\mu \cdot 2) \cdot \left[(5 - 2.5) + \sqrt{\frac{2 \cdot 100\mu}{92\mu \cdot 5}}\right] \approx 580\mu S$$

b) Simply use the values: $i_o = G_m \cdot (v_i^+ - v_i^-) = G_m \cdot (v_i^+ - 2.5)$

c) We cannot allow the approximation of the current through Q_9. In this case

$$i_{D,9} = \frac{\beta_9}{2} \cdot (2(v_{GS,9} - v_{T,9}) - v_{DS,9}) \cdot v_{DS,9}$$

By differenting we find

$$G_m = g_{ds} = \frac{\partial i_{D,9}}{\partial v_{DS,9}} = \beta_9 \cdot (v_{GS,9} - v_{T,9} - v_{DS,9}) =$$
$$= \beta_9 \cdot [(V_C - v_{S,1}) - v_{T,12} + (v_i^+ - v_i^-)] = \beta_9 \cdot [(V_C - v_i^-) + \sqrt{2I_1/\beta_1}]$$

etc.

Exercise B15.12

$(W/L) = 10/2 = 5$, $V_{DD} = -V_{SS} = 2.5\,\text{V}$, and $V_{C,1} = -V_{C,2} = 2\,\text{V}$

We first have to consider the CMOS pair that is described on pages 608-609. When cascoding a NMOS and PMOS transistor with same drain current flowing through both devices we can consider them as one transistor with certain properties. Consider

$$i_{D,n} = K_n \cdot (v_{GS,n} - v_{T,n})^2 \text{ and } i_{D,p} = K_p \cdot (v_{SG,p} + v_{T,p})^2$$

We see that

$$v_{GS,n} = v_{T,n} + \sqrt{i_D/K_n} \text{ and } v_{SG,p} = -v_{T,p} + \sqrt{i_D/K_p} \text{ where } i_{D,n} = i_{D,p} = i_D$$

Let

$$v_{GS,pn} = v_{GS,n} + v_{SG,p} = (v_{T,n} - v_{T,p}) + \frac{\sqrt{i_D}}{\sqrt{K_n} + \sqrt{K_p}}$$

Let further

$$v_{T,pn} = v_{T,n} - v_{T,p} \text{ and } \frac{1}{\sqrt{K_{pn}}} = \frac{1}{\sqrt{K_n} + \sqrt{K_p}}$$

hence

$$v_{GS,pn} = v_{T,pn} + \sqrt{i_D/K_{pn}}$$

We now see that the drain current can be written as

$$i_D = K_{pn} \cdot (v_{GS,pn} - v_{T,pn})^2$$

In figure 15.30

$$i_1 = K_{pn} \cdot (V_{C,1} - v_{in} - v_{T,pn})^2$$
$$i_2 = K_{pn} \cdot (v_{in} - V_{C,2} - v_{T,pn})^2 = K_{pn} \cdot (v_{in} + V_{C,1} - v_{T,pn})^2$$
$$i_1 - i_2 = K_{pn} \cdot (V_{C,1}^2 + v_{in}^2 + v_{T,pn}^2 - 2V_{C,1}v_{in} - 2V_{C,1}v_{T,pn} + 2v_{in}v_{T,pn}) -$$
$$-K_{pn} \cdot (v_{in}^2 + V_{C,1}^2 + v_{T,pn}^2 + 2V_{C,1}v_{in} - 2V_{C,1}v_{T,pn} - 2v_{in}v_{T,pn}) =$$
$$= K_{pn} \cdot (4V_{C,1}v_{in} - 4v_{T,pn}v_{in}) = 4K_{pn} \cdot (V_{C,1} - v_{T,pn}) \cdot v_{in}$$

Then the G_m is given by

$$G_m = \frac{i_1 - i_2}{v_{in}} = 4K_{pn} \cdot (V_{C,1} - v_{T,pn})$$

Use the values from page 78 to find the result.