Lesson 9

Lesson Exercises: **B10.5 - B10.10, K31, K32, K33, K34, K40**

Recommended Exercises: *K29, K35*

Theoretical Issues: **SC-filter**

Theoretical

• SC-filter

Leapfrogfilter

In many cases we use a continous-time analog reference filter to find the specifications on the SC filter. The filter is transformed with LDI or bilinear transformation into a suitable discrete-time representation.

Lossless Discrete Integrator, LDI transformation

Let the transformation be given by

$$s = s_0[z^{1/2} - z^{-1/2}] = s_0\frac{1-z^{-1}}{z^{-1/2}} = s_0\frac{z-1}{z^{1/2}} = s_0 \cdot \frac{1-z^{-1}}{z^{-1/2}}$$

Or using integration notation

$$\frac{1}{s} = \frac{1}{s_0} \cdot \frac{z^{-1/2}}{1-z^{-1}}$$

Let

$$s = j\omega \text{ and } z = e^{j\Omega}$$

which gives

$$\omega = 2s_0\sin\left(\frac{\Omega}{2}\right) \text{ or } s_0 = \frac{\omega}{2\sin(\Omega/2)}$$

where ω is the continous-time angular frequency and Ω is the discrete-time. Due to the LDI mapping we see that

$$\omega < 2s_0$$

From that we conclude that the filters to be transformed must be narrow banded. This is a drawback with the LDI transformation.

We also find that

$$z^{-1/2} = e^{-j\Omega/2} = \cos\frac{\Omega}{2} - j \cdot \sin\frac{\Omega}{2}$$

Example Exercise K40

Synthesize an LDI filter. Transformation give

$$s = s_0\frac{z-1}{z^{1/2}}$$

The specification on the reference filter angular cut-off frequency is chosen to be

$$\omega_c = 2\pi \cdot 1500 \text{ rad/s}$$

Since sampling frequency is $f_s = 40\text{ kHz}$ we have the discrete-time cut-off frequency as

$$\Omega_c = \frac{1.5k}{40k} \cdot 2\pi \approx 0.2356$$

and the discrete-time stop-band frequency as

$$\Omega_s = \frac{2.5k}{40k} \cdot 2\pi \approx 0.3927$$

From this we have s_0

$$s_0 = \frac{\omega_c}{2\sin(\Omega_c/2)} =$$

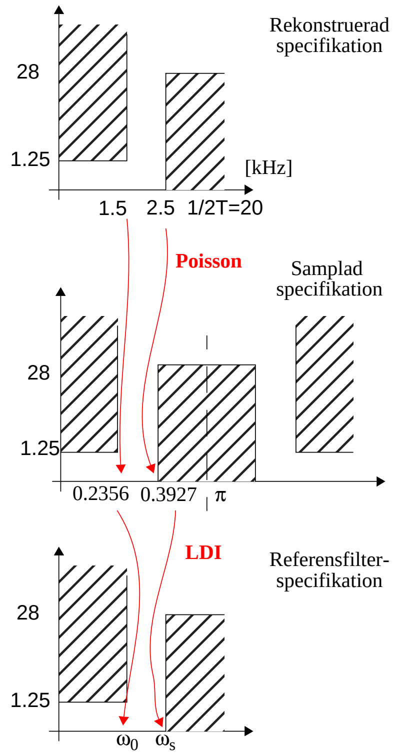
$$= \frac{3000\pi}{2\sin(0.2356/2)} \approx 40.093 \text{ krad/s}$$

We find the ω_s

$$\omega_0 = 2s_0\sin(\Omega_s/2) \approx$$

$$\approx 2 \cdot 40.093 \cdot \sin(0.3927/2) \approx$$

$$\approx 15.643 \text{ krad/s}$$



Design an elliptic filter. Order is found to be $N = 3$. Suppose the resistors are equal, or

$$\kappa^2 = 1 \text{ and choose } R_i = R_0 = 1k\Omega$$

The normated values on the components are

$$C_{1n} = C_{3n} = 1.9314,\ C_{2n} = 0.3781 \text{ and } L_{2n} = 0.7571$$

These are denormalized with

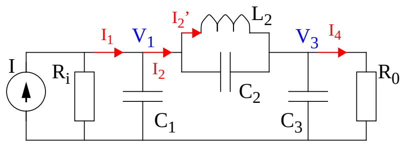
$$L_i = \frac{R_0}{\omega_0}L_{in} \text{ and } C_i = \frac{1}{\omega_0 R_0}C_{in}$$

Which gives

$$C_1 = C_3 = 204.9nF$$

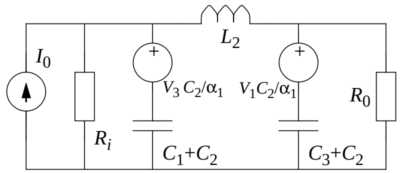
$$L_2 = 80.3mH$$

$$C_2 = 40.1nF$$



The filter is transformed. The constant used in the figure is given by

$$\alpha_1 = C_1 + C_2 = C_2 + C_3$$



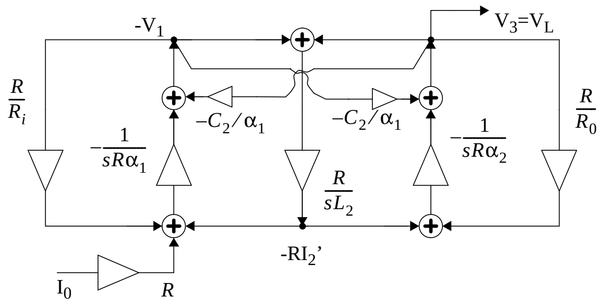
Setting up the wellknown equations for currents and voltages in the filter, we have

$$RI_0 = RI_i - \frac{R}{R_i}V_1$$

$$V_1 = \frac{1}{sRC_1}(RI_0 - RI_2),\; RI_2 = \frac{R}{(sL_2)\,\|\,(1/sC_2)}(V_1 - V_3)$$

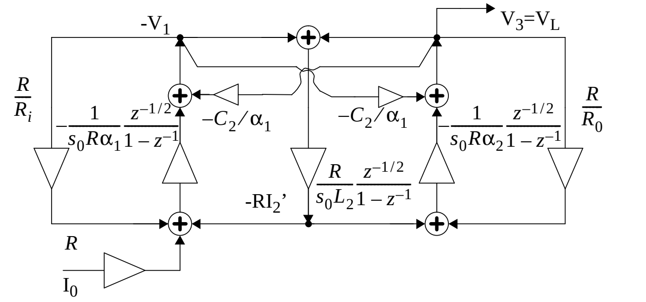
$$V_3 = \frac{1}{sRC_3}(RI_2 - RI_4),\; RI_4 = \frac{R}{R_0}V_3 = \frac{R}{R_0}V_L$$

etc. With this the signal flow graph becomes

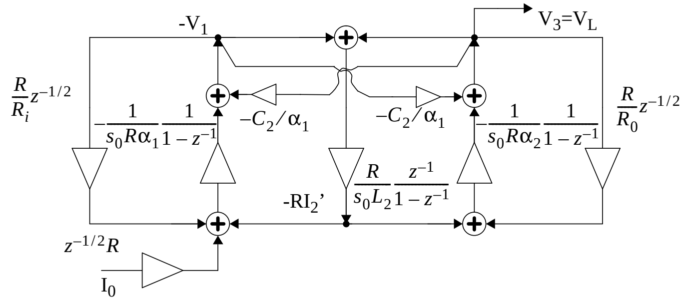


LDI transform by setting

$$s = s_0 \cdot \frac{z-1}{z^{1/2}}$$



Elminate $z^{-1/2}$ in the integrators by propagating backwards.



We now though have $z^{-1/2}$ terms in the outer feedback amplifiers. This is practically not possible to implement. One way to implement this is simply to remove the $z^{-1/2}$ term in the expression:

$$\frac{R}{R_L} \cdot z^{-1/2}$$

We could assume that the original R_L and R_i have the expressions

$$R_L = R_L \cdot z^{-1/2} \text{ and } R_i = R_i \cdot z^{-1/2}$$

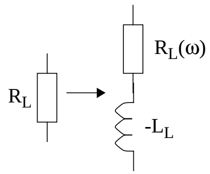
As discussed earlier, $z^{-1/2}$ naturally contains valuable frequency information

$$z^{-1/2} = -j\frac{\omega}{2s_0} + \sqrt{1 - \left(\frac{\omega}{2s_0}\right)^2}$$

Thereby

$$R_L \cdot z^{-1/2} = R_L\left[-j\frac{\omega}{2s_0} + \sqrt{1-\left(\frac{\omega}{2s_0}\right)^2}\right] =$$

$$= R_L\sqrt{1-\left(\frac{\omega}{2s_0}\right)^2} - j\omega\frac{R_L}{2s_0} = R_L(\omega) - j\omega L_L$$

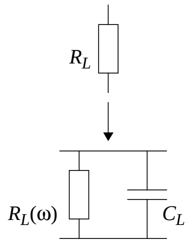


Which is realized by a frequency dependent resistance in series with an inductance, or more useful in this filter implemenation, as a frequency dependet resistance in parallel with a capacitor.

$$R_L \cdot z^{-1/2} = \frac{(R_L(\omega) - j\omega L_L)(R_L(\omega) + j\omega L_L)}{R_L(\omega) + j\omega L_L} =$$

$$= \frac{R_L^2 \cdot 1}{R_L\sqrt{1-(\omega/2s_0)^2} + j\omega R_L/2s_0} =$$

$$= \frac{1}{\dfrac{1}{R_L/\sqrt{1-(\omega/2s_0)^2}} + \dfrac{1}{R_L/(j\omega/2s_0)}} = R_L(\omega) \parallel C_L$$

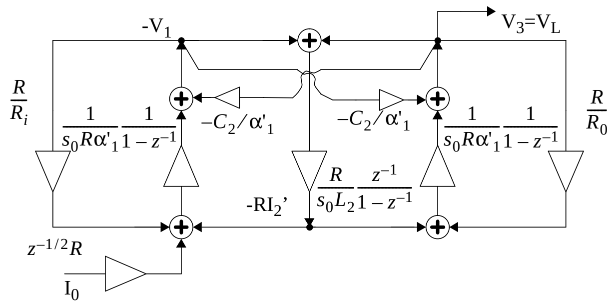


Suppose that C_L is coupled in parallel with C_3 and correspondingly for the inner source resistance C_i is in parallel with C_1. This is corrected by letting the components have the values

$$C_1' = C_1 - C_i = C_1 - \frac{1}{2s_0R_i} = C_1 - \frac{1}{\omega_0 R_i}\sin\left(\frac{\Omega_0 T}{2}\right) \text{ and}$$

$$C_3' = C_3 - C_L = C_3 - \frac{1}{2s_0R_L} = C_3 - \frac{1}{\omega_0 R_L}\sin\left(\frac{\Omega_0 T}{2}\right)$$

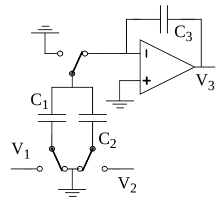
We still have an error that is caused by the fact that we will not implement a frequency dependent resistance. This error is considered to be acceptable. The realization of the filter is given by the flow graph



The two integrators (inverting amplifiers with and without delay) are replaced with their corresponding SC circuits.

The transfer function of the summing integrator is given by

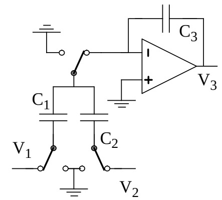
$$V_3(z) = \frac{z^{-1}}{1-z^{-1}} \cdot \left[\frac{C_1}{C_3} \cdot V_1(z) + \frac{C_2}{C_3} \cdot V_2(z)\right]$$



Check: No direct signal path from input to the output.

The transfer function of the summing and inverting integrator is given by

$$V_3(z) = -\frac{1}{1-z^{-1}} \cdot \left[\frac{C_1}{C_3} \cdot V_1(z) + \frac{C_2}{C_3} \cdot V_2(z)\right]$$



Check: Direct signal path from input to the output.

(Charge that is directed to C_3 is given by a linear combination of the input signals $v_1(t)$ and $v_2(t)$.)

Integrators are used in the realization. The sizes of the capacitors have to be identified. This is done by comparing signal paths in the SC realization with those of the signal flow graph:

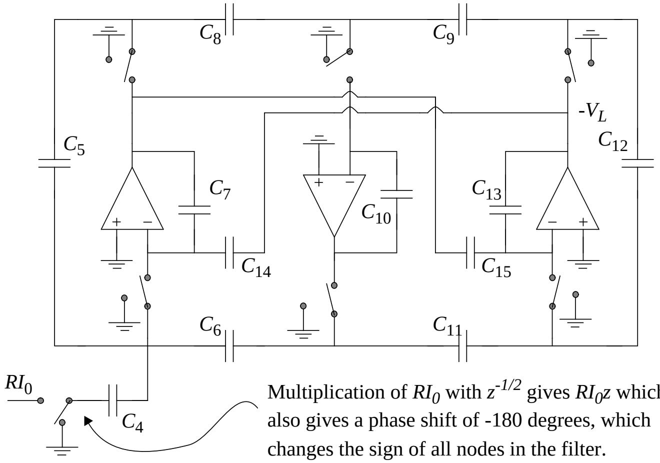
SC filter	Signal Flow Graph	Result
$[-V_1]_E = -\frac{C_4}{C_7}\frac{z^{-1/2}}{1-z^{-1}}$	$[-V_1]_E = -\frac{1}{s_0R\alpha_1'} \cdot \frac{z^{-1/2}}{1-z^{-1}}$	$\frac{C_4}{C_7} = \frac{1}{s_0R\alpha_1'}$
$[-V_1]_{-RI_2'} = -\frac{C_6}{C_7}\frac{1}{1-z^{-1}}$	$[-V_1]_{-RI_2'} = -\frac{1}{s_0R\alpha_1'}$	$\frac{C_6}{C_7} = \frac{1}{s_0R\alpha_1'}$
$[-V_1]_{-V_1} = -\frac{C_5}{C_7}\frac{1}{1-z^{-1}}$	$[-V_1]_{-V_1} = -\frac{1}{s_0R\alpha_1'}\frac{R}{R_i} = -\frac{1}{s_0R_i\alpha_1'}$	$\frac{C_5}{C_7} = \frac{1}{s_0R_i\alpha_1'}$
$[-RI_2']_{-V_1} = \frac{C_8}{C_{10}}\frac{z^{-1}}{1-z^{-1}}$	$[-RI_2']_{-V_1} = \frac{R}{s_0L_2}\frac{z^{-1}}{1-z^{-1}}$	$\frac{C_8}{C_{10}} = \frac{R}{s_0L_2}$
$[-RI_2']_{V_3} = \frac{C_9}{C_{10}}\frac{z^{-1}}{1-z^{-1}}$	$[-RI_2']_{V_3} = \frac{R}{s_0L_2}\frac{z^{-1}}{1-z^{-1}}$	$\frac{C_9}{C_{10}} = \frac{R}{s_0L_2}$
$[V_3]_{-RI_2'} = -\frac{C_{11}}{C_{13}} \cdot \frac{1}{1-z^{-1}}$	$[V_3]_{-RI_2'} = -\frac{R}{R_L} \cdot \frac{1}{s_0RC_3} \cdot \frac{1}{1-z^{-1}}$	$\frac{C_{11}}{C_{13}} = \frac{1}{s_0R_L\alpha_3'}$
$[V_3]_{V_3} = -\frac{C_{12}}{C_{13}}\frac{1}{1-z^{-1}}$	$[V_3]_{V_3} = -\frac{R}{R_L} \cdot \frac{1}{s_0RC_3} \cdot \frac{1}{1-z^{-1}}$	$\frac{C_{12}}{C_{13}} = \frac{1}{s_0R_L\alpha_3'}$

For the feedback we have:

$[-V_1]_{V_3} = -\frac{C_{14}}{C_7}$	$[-V_1]_{V_3} = -\frac{C_2}{\alpha_1'}$	$\frac{C_{14}}{C_7} = \frac{C_2}{\alpha_1'}$
$[V_3]_{-V_1} = -\frac{C_{15}}{C_{13}}$	$[V_3]_{-V_1} = -\frac{C_2}{\alpha_3'}$	$\frac{C_{15}}{C_{13}} = \frac{C_2}{\alpha_3'}$

We assume that

$$R_i = R_L = R$$



Multiplication of RI_0 with $z^{-1/2}$ gives $RI_0 z$ which also gives a phase shift of -180 degrees, which changes the sign of all nodes in the filter.

Further, the correction term gives

$$s_0 = \frac{\omega_c}{2\sin(\Omega_c/2)} \text{ and } C_i' = C_i - 1/2s_0R$$

Which yields

$$\frac{C_4}{C_7} = \frac{C_5}{C_7} = \frac{C_6}{C_7} = \left[\frac{\omega_c R(C_1+C_2)}{2\sin(\Omega_c/2)} - \frac{R}{2R}\right]^{-1}, \frac{C_{11}}{C_{13}} = \frac{C_{12}}{C_{13}} = \left[\frac{\omega_c R(C_3+C_2)}{2\sin(\Omega_c/2)} - \frac{R}{2R}\right]^{-1}$$

$$\frac{C_8}{C_{10}} = \frac{C_9}{C_{10}} = \frac{2R\sin(\Omega_c/2)}{\omega_c L_2}$$

$$\frac{C_{14}}{C_{13}} = \frac{C_2}{C_2 + C_1 - \frac{\sin(\Omega_c/2)}{R_i\omega_c}} \text{ and } \frac{C_{15}}{C_{13}} = \frac{C_2}{C_2 + C_3 - \frac{\sin(\Omega_c/2)}{R_L\omega_c}}$$

With values we have

$$\frac{C_4}{C_7} = \frac{C_5}{C_7} = \frac{C_6}{C_7} = \frac{C_{11}}{C_{13}} = \frac{C_{12}}{C_{13}} = \left[\frac{2\pi \cdot 1500 \cdot 1000 \cdot (204.9n + 40.1n)}{2\sin(0.2356/2)} - \frac{1}{2}\right]^{-1} \approx 0.1072$$

$$\frac{C_8}{C_{10}} = \frac{C_9}{C_{10}} = \frac{2 \cdot 1000 \cdot \sin(0.2356/2)}{3000\pi \cdot 80.3m} \approx 0.3106$$

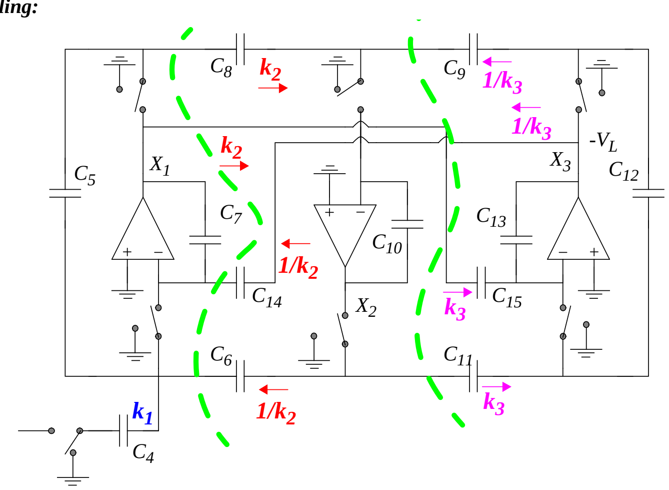
$$\frac{C_{14}}{C_{13}} = \frac{C_{15}}{C_{13}} = \frac{40.1n}{40.1n + 204.9n - \frac{\sin(0.2356/2)}{1000 \cdot 3000\pi}} \approx 0.1725$$

Choose the integrators' capacitors all equal

$$C_7 = C_{10} = C_{13} = 47\,\text{nF}$$

From this we have the values of all other capacitors.

Scaling:

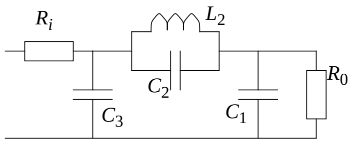


Scale the filter so that the relation between the OPamp outputs and the input signal is unity (L_∞-norm). We are scaling the filter to keep the signal levels at wanted levels. The principle of scaling can be described by dividing the net into subnets. The subnets have a number of inputs and outputs. If the inputs are scaled with a constant k_i all nodes of the subnet will be scaled with a factor k_i, as well as we have to scale all outputs with $1/k_i$.

In this case the signal after the first node is scaled to k_1X_1, the second with $k_2k_1X_2$ and finally the third (the output) with $k_3k_2k_1X_3$. By changing the values of the capacitances we now can realize the scaling. At C_4 we use k_1C_4 instead. For $C_6 \to C_6/k_2$, $C_8 \to C_8 \cdot k_2$, $C_{14} \to \frac{C_{14}}{k_2k_3}$, $C_{15} \to C_{15} \cdot k_2k_3$, $C_9 \to C_9/k_3$, $C_{11} \to C_{11} \cdot k_3$ etc.

Exercises

Exercise K35



Third order low pass filter with an elliptic reference filter. The specification gives

$f_c = 3.4kHz$, $A_{max} = 0.02\,\text{dB}$,

$f_{sample} = 128kHz$,

Normalized values from table are:

$$R_i = R_0 = 1k\Omega,\ C_{1,n} = C_{3,n} = 0.5275,\ C_{2,n} = 0.1921,\ L_{2,n} = 0.7700$$

LDI transformation gives

$$s = s_0 \cdot \frac{1 - z^{-1}}{z^{-1/2}} \text{ where } s_0 = \frac{\omega_{ac}}{2\sin(\omega_c T/2)}$$

Denormalize the values.

Compensate for the LDI transformation errors.

Find the flow graph

Use standard SC integrators.

Identify the values

Exercise K34

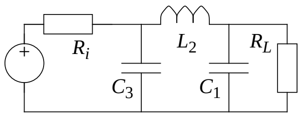
Due to the fact that

$R = R_i = R_L$

we know that the dc gain is $1/2$

You can also see that the filter is realizing a third order elliptic low pass filter. Suppose that the maximum output value is given by dc voltage. This implies that we directly can choose the scaling parameter k_1 in such a way that all nodes in the net become scaled with a factor two, hence $k_1 = 2$.

Exercise K33



The order is found to be $N = 3$. Values are

$C_{3n} = 1$, $L_{2n} = 2$, $C_{1n} = 1$ and

$R_i = R_L = 1k\Omega$