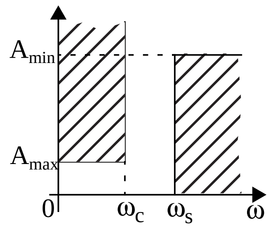
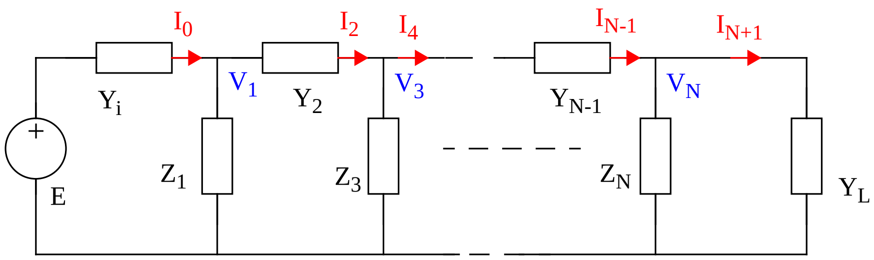
1 Leapfrog Filter Synthesis — Extra info



1. Use the filter specification. Peform needed transformatioms (BP/HP/LP/BS).

2. Identify the filter order and component values for the passive filter implementation. Find the ladder structure. Unnormalize the component values.

3. Introduce currents and voltages in the ladder network.



4. Find the equations determining the relation between the I_i and V_i.

   $I_0 = Y_i(E - V_1)$, $V_1 = Z_1(I_0 - I_2)$, ...

   $I_{2k} = Y_{2k}(V_{2k-1} - V_{2k+1})$, $V_{2k+1} = Z_{2k+1}(I_{2k} - I_{2(k+1)})$, ...

   $I_{N+1} = Y_L(V_N - 0)$, $V_N = V_L$ (dependent on the filter order)

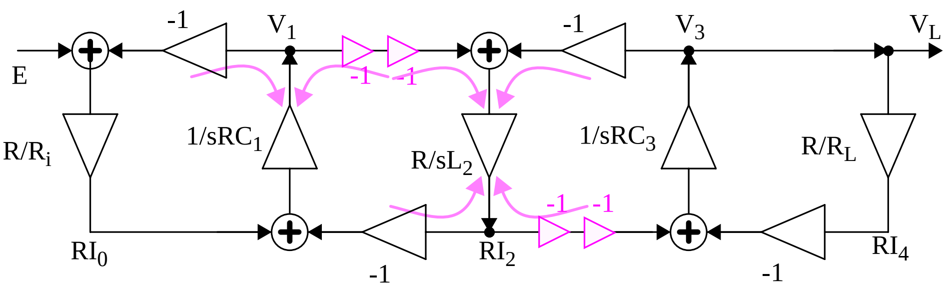
5. Normalize all equations with the constant R to achieve voltage-unit variables

$$RI_0 = RY_i(E - V_1),\ V_1 = \frac{Z_1}{R}(RI_0 - RI_2),\ \ldots$$

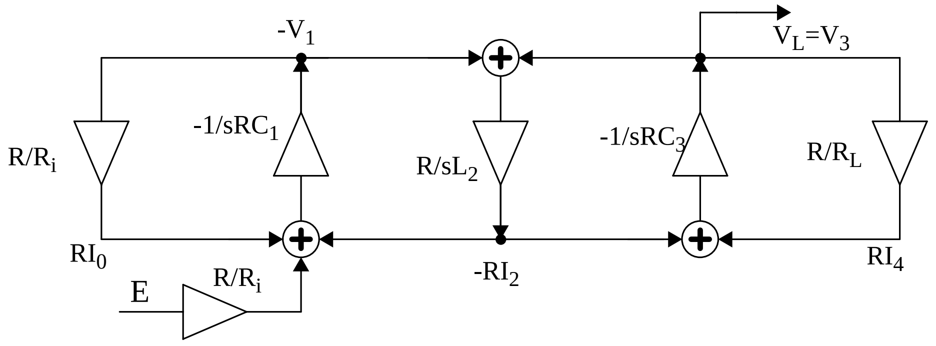
$$RI_{2k} = RY_{2k}(V_{2k-1} - V_{2k+1}),\ V_{2k+1} = \frac{Z_{2k+1}}{R}(RI_{2k} - RI_{2(k+1)}),\ \ldots$$

$$RI_{N+1} = RY_L(V_N - 0),\ V_N = V_L \text{ (dependent on the filter order)}$$

6. Create the signal flow graph describing the equation system. Example for a third order filter:



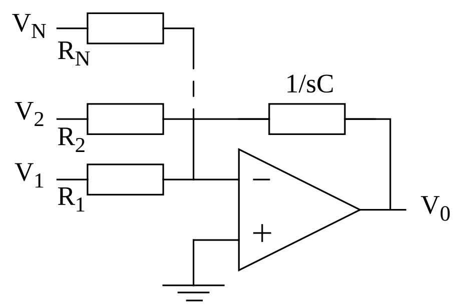
Modify the graph by propagating the -1's through the net and notify with negative potentials, e.g., $-V_1$:



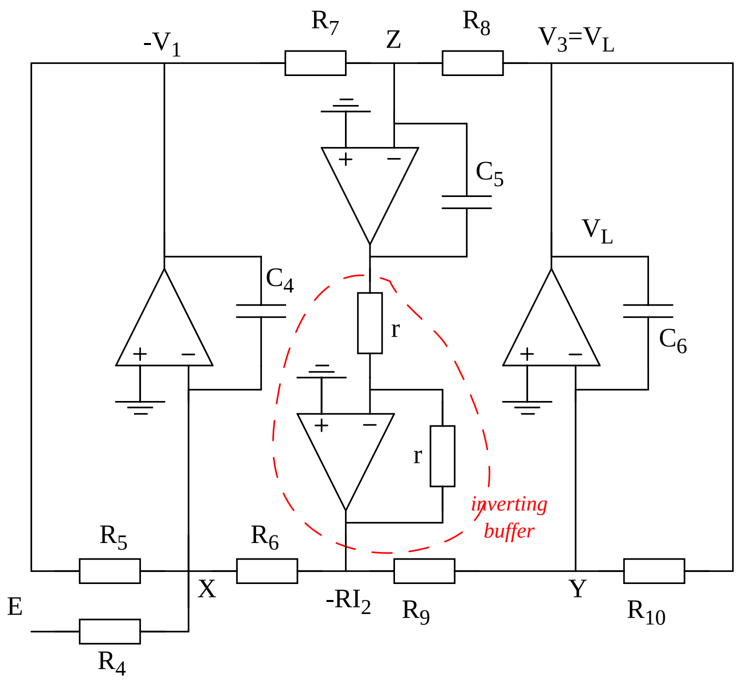
In the filter, we have three amplifying and summating integrators: $-1/sRC_1$, R/sL_2, and $-1/sRC_3$.

A summating integrator can be implemented using active components. The transfer function for this integrator becomes:

$$V_0 = -\frac{1}{sC}\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \ldots + \frac{V_N}{R_N}\right)$$



This is a inverting integrator. This implies that the integrator R/sL_2 has to use an inverting buffer to achieve a proper signal path. This gives the following structure:



Now, the component values have to be identified. This is done by comparing the (original) signal flow graph with the active network.

For example:

Active Leapfrog	Signal flowgraph	Result
$[-V_1]_E = -\frac{1}{sC_4} \cdot \frac{E}{R_4}$	$[-V_1]_E = \frac{R}{R_i} \cdot -\frac{E}{sRC_1} = -\frac{E}{sR_iC_1}$	$C_4R_4 = C_1R_i$
$[-V_1]_{-V_1} = -\frac{1}{sC_4} \cdot \frac{V_1}{R_5}$	$[-V_1]_{-V_1} = \frac{R}{R_i} \cdot -\frac{-V_1}{sRC_1} = -\frac{-V_1}{sR_iC_1}$	$C_4R_5 = C_1R_i$
$[-V_1]_{-RI_2} = -\frac{1}{sC_4}\frac{-RI_2}{R_6}$	$[-V_1]_{-RI_2} = -\frac{1}{sRC_1}(-RI_2)$	$C_4R_6 = C_1R$
$[-RI_2]_{-V_1} = -\frac{(-1)}{sC_5} \cdot \frac{-V_1}{R_7}$	$[-RI_2]_{-V_1} = \frac{R}{sL_2}(-V_1)$	$C_5R_7 = L_2/R$
$[-RI_2]_{V_3} = -\frac{(-1)}{sC_5} \cdot \frac{V_3}{R_8}$	$[-RI_2]_{V_3} = \frac{R}{sL_2}V_3$	$C_5R_8 = L_2/R$
$[V_3]_{-RI_2} = -\frac{1}{sC_6}\frac{-RI_2}{R_9}$	$[V_3]_{-RI_2} = -\frac{1}{sRC_3}(-RI_2)$	$C_6R_9 = C_3R$
$[V_3]_{V_3} = -\frac{1}{sC_6}\frac{V_3}{R_{10}}$	$[V_3]_{V_3} = -\frac{1}{sRC_3}\frac{R}{R_L}V_3 = -\frac{V_3}{sC_3R_L}$	$C_6R_{10} = C_3R_L$

In order to achieve good matching properties, we choose the capacitances to be equally large. For example:

$$C_4 = C_5 = C_6 = 30nF$$

(This may be somewhat too large in an integrated circuit). From the equations above, we see that

$$R_4 = R_5 = \frac{C_1R_i}{C_4} = \frac{36.3nF \cdot 1k\Omega}{30nF} = 1.21k\Omega \text{ och } R_{10} = \frac{C_3R_L}{C_6} = 1.21k\Omega$$

and

$$R_6 = R_9 = \frac{C_1R}{C_4},\ R_7 = R_8 = \frac{L_2}{C_5R}$$

For symmetry and hence proper matching, we choose $R_6 = R_7 = R_8 = R_9$, and

$$R^2 = \frac{L_2C_4}{C_1C_5} \Rightarrow R = \sqrt{L_2/C_1} = 1000\sqrt{72.6/36.3} \approx 1.41k\Omega$$

Finally

$$R_6 = R_7 = R_8 = R_9 = 1.21 \cdot 1.41k\Omega \approx 1.71k\Omega$$

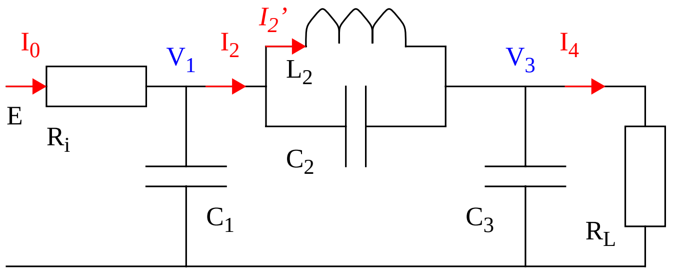
The resistance values of the inverting buffer, r, also have to be determined. Choose (for matching) for example

$$r = R_6 = 1.71k\Omega$$

Note that R_4 can be chosen so that the dc gain is 1. This is done by reducing R_4 to half its value (scaling of the entire filter).

Special case: Cauer Filters

Example: Consider the Cauer filter.



The equations for the voltages and currents are (unnormalized)

$$I_0 = \frac{E - V_1}{R_i},\ I_2 = \frac{1}{L_2 \parallel C_2}(V_1 - V_3),\ I_4 = \frac{V_3}{R_L},$$

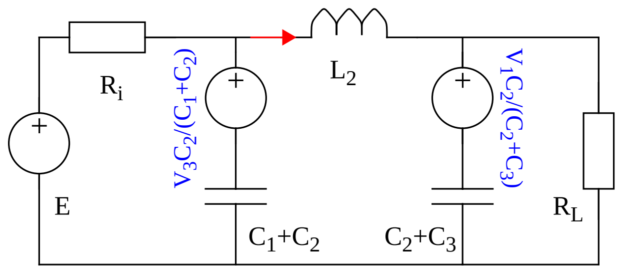
$$V_1 = \frac{1}{sC_1}(I_0 - I_2),\ \text{and } V_3 = \frac{1}{sC_3}(I_2 - I_4).$$

For voltage-unit variables, we have normalized with R:

$$RI_0 = \frac{R}{R_i}(E - V_1),\ RI_2 = \frac{R}{sL_2/(1 + s^2L_2C_2)}(V_1 - V_3),\ RI_4 = \frac{R}{R_L}V_3,$$

$$V_1 = \frac{1}{sRC_1}(RI_0 - RI_2),\ \text{and } V_3 = \frac{1}{sRC_3}(RI_2 - RI_4).$$

The inductance and capacitance in parallel introduces some complexity to the signal flow graph. The capacitance C_2 can be eliminated by introducing voltage dependent sources to the circuit.



Examin the help current, I'_2. Now we can rewrite the equations as

$$RI_2 = RI_2' + sRC_2(V_1 - V_3)\ \text{and } RI_2' = \frac{R}{sL_2}(V_1 - V_3).$$

This gives

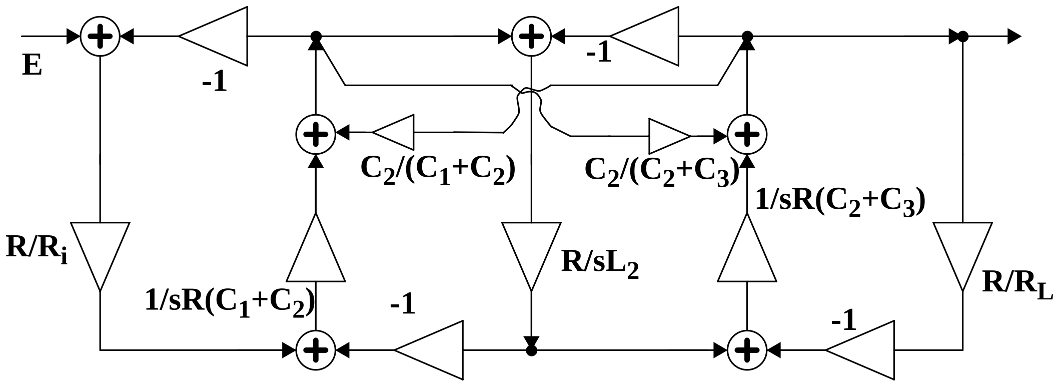
$$V_1 = \frac{1}{sRC_1}(RI_0 - RI_2' - sRC_2(V_1 - V_3)) \Rightarrow$$

$$V_1 = \frac{1}{sR(C_1 + C_2)}(RI_0 - RI_2') + \frac{C_2}{C_1 + C_2}V_3$$

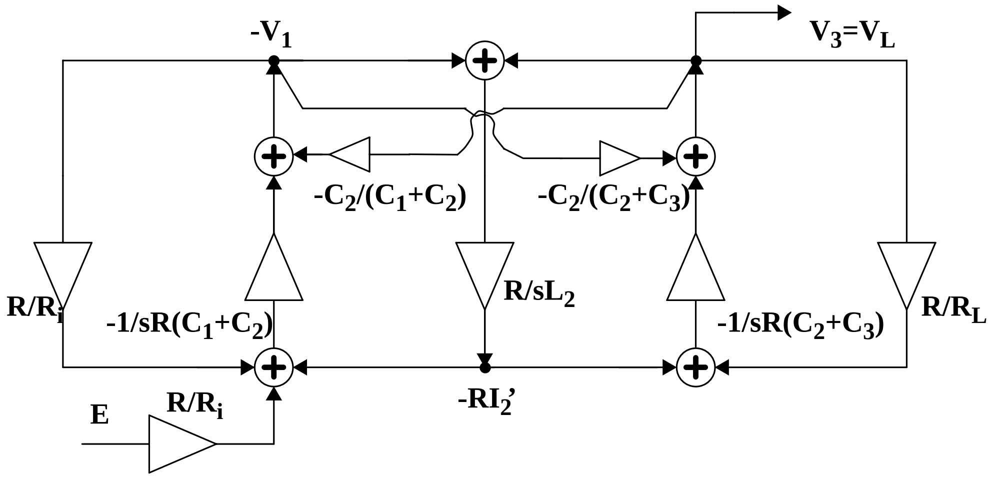
And similar

$$V_3 = \frac{1}{sR(C_2 + C_3)}(RI_2' - RI_4) + \frac{C_2}{C_2 + C_3}V_1$$

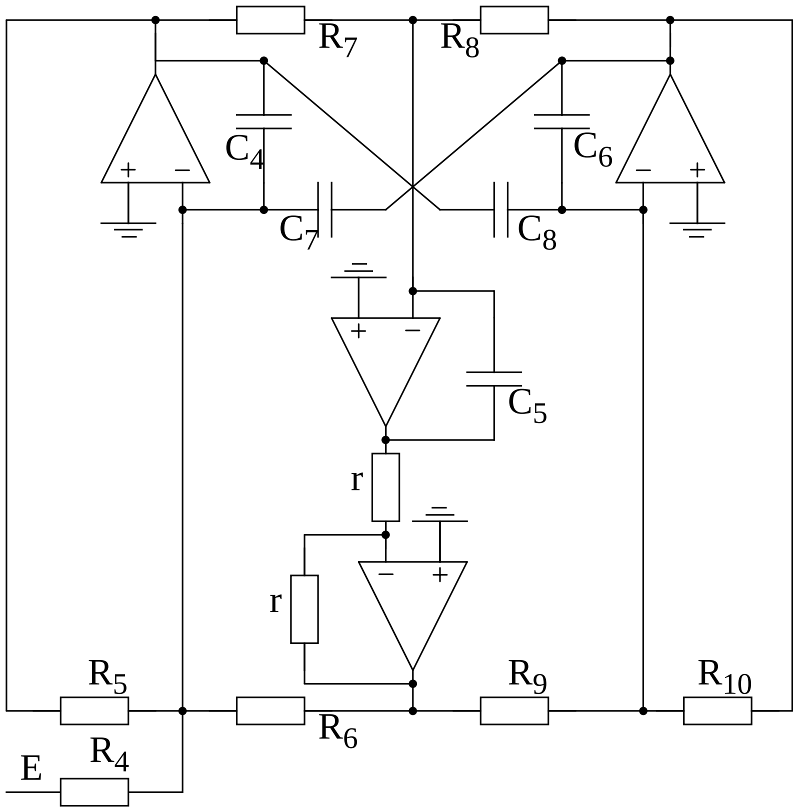
The corresponding signal flow graph becomes:



The −1 ‘s are eliminated:



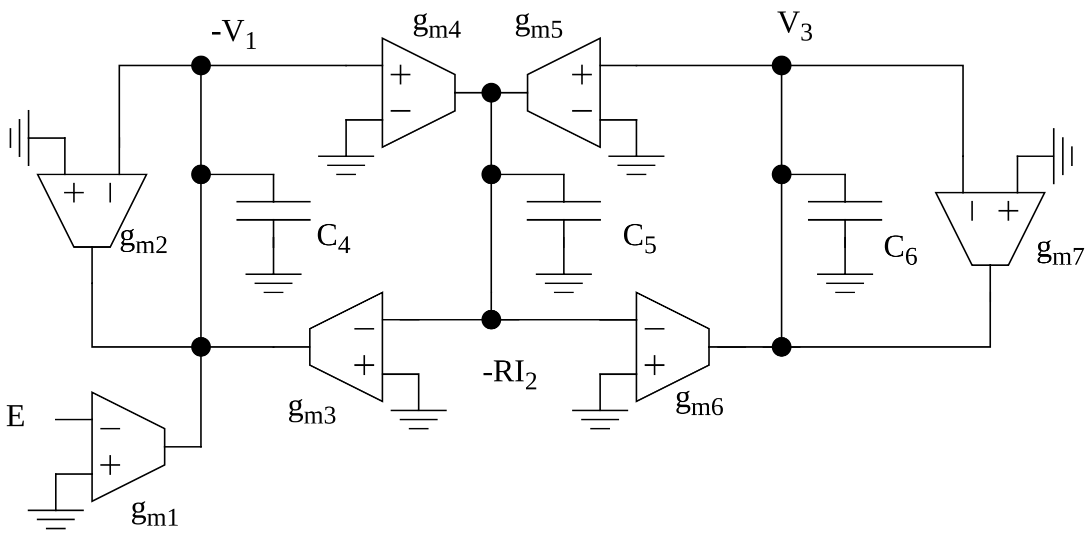
The active filter implementation becomes:



Note the use of C_7 and C_8 as summation elements in the integrators. Component values are identified as in the previous case.

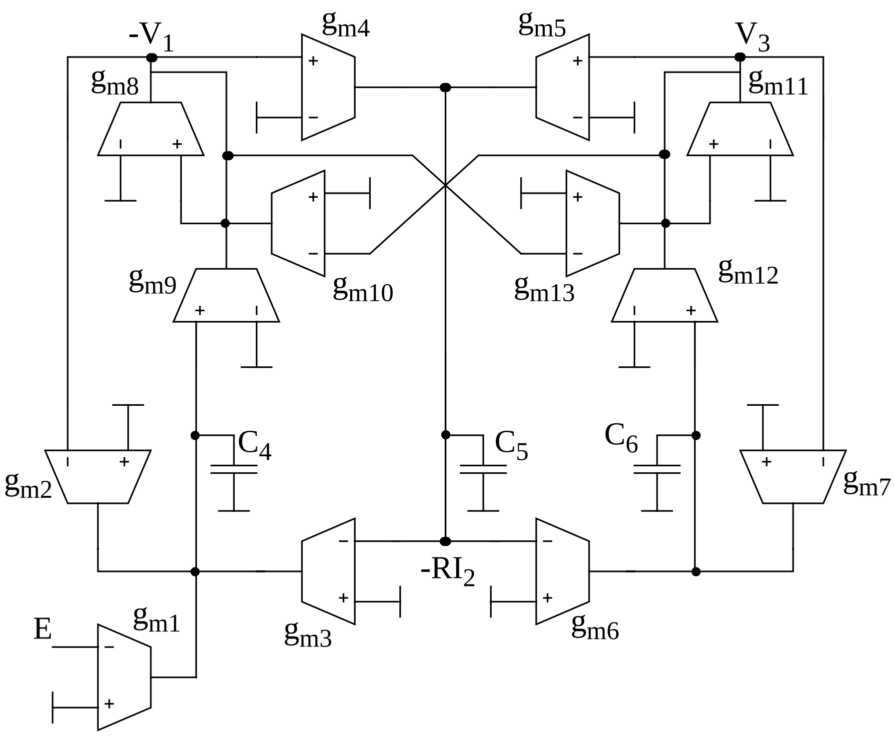
Gm-C implementasttion (Non-elliptic)

The same signal-flow graph is used, but now we insert the corresponding integrating and summating elements described by Gm-C building blocks.



Gm-C	Signal flow graph	Result
$[-V_1]_E = -\frac{g_{m1}}{sC_4}\cdot E$	$[-V_1]_E = \frac{R}{R_i}\cdot -\frac{E}{sRC_1} = -\frac{E}{sR_iC_1}$	$C_4 = g_{m1}C_1R_i$
$[-V_1]_{-V_1} = -\frac{g_{m2}}{sC_4}\cdot V_1$	$[-V_1]_{-V_1} = -\frac{-V_1}{sRC_1}\frac{R}{R_i} = -\frac{-V_1}{sR_iC_1}$	$C_4 = g_{m2}C_1R_i$
$[-V_1]_{-RI_2} = -\frac{g_{m3}}{sC_4}(-RI_2)$	$[-V_1]_{-RI_2} = -\frac{1}{sRC_1}(-RI_2)$	$C_4 = g_{m3}C_1R$
$[-RI_2]_{-V_1} = \frac{g_{m4}}{sC_5}\cdot -V_1$	$[-RI_2]_{-V_1} = \frac{R}{sL_2}(-V_1)$	$C_5 = g_{m4}L_2/R$
$[-RI_2]_{V_3} = \frac{g_{m5}}{sC_5}\cdot V_3$	$[-RI_2]_{V_3} = \frac{R}{sL_2}V_3$	$C_5 = g_{m5}L_2/R$
$[V_3]_{-RI_2} = -\frac{g_{m6}}{sC_6}(-RI_2)$	$[V_3]_{-RI_2} = -\frac{1}{sRC_3}(-RI_2)$	$C_6 = g_{m6}C_3R$
$[V_3]_{V_3} = -\frac{g_{m7}}{sC_6}V_3$	$[V_3]_{V_3} = \frac{-1}{sRC_3}\frac{R}{R_L}V_3 = \frac{-V_3}{sC_3R_L}$	$C_6 = g_{m7}C_3R_L$

Special case: Cauer Filters



With: $\frac{g_{m9}}{g_{m10}} = \frac{C_2}{\alpha_1} = \frac{C_2}{C_1 + C_2}$ and $\frac{g_{m9}}{g_{m8}} = 1$