5 Filters

Part 5.A—Filter specification

A specification on a bandstop (BS) filter is shown in Fig. 5.1 where the attenuation

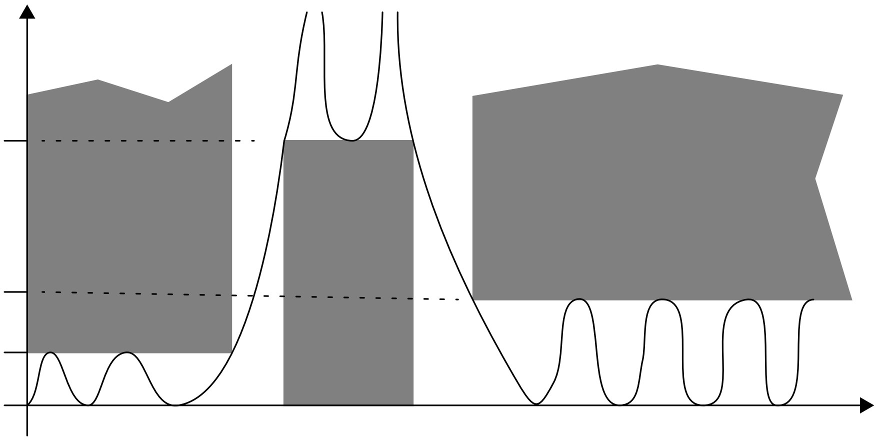

Figure 5.1: A typical filter specification for a bandstop (BS) filter.

function is given. The attenuation is found by

$$A(j\omega) = 20\log\left|\frac{H_0}{H(j\omega)}\right| \text{ and } A(j\omega) = -20\log|H(j\omega)| \text{ when } H_0 = 1$$

Where the transfer function is defined as

$$H(s) = \frac{X(s)}{Y(s)}$$

The specification in the figure above is describing a bandstop (BS) filter. The stopband limits are given by ω_{sl} and ω_{sh}. The minimum allowed attenuation in the passband is given by A_{min}. In the lower passband, we have $A_{max,\,l}$ and ω_{pl}, and in the upper passband, $A_{max,\,h}$ and ω_{ph}.

However, mostly we consider the low pass filter in our applications (and especially in this course). This implies that the upper passband dissappears. We use parameters such as the

Attenuation (or ripple) in pass- and stopband: A_{max} and A_{min}

Passband and stopband edges: ω_p and ω_s

The ripple in the passband is also given by

$$A_{max} = 10\log(1 + \varepsilon^2)$$

We will not go into details about filters since this is the topics of other courses. However, roughly, the filter characteristics can be given by Butterworth, Cauer, or Chebyshev polynomials. We characterize these in a coarse way and say that

- Butterworth filter has maximally flat stopband and passband
- Cauer filter has equiripple in stop- and passband

- Chebyshev I filter has equiripple in the passband and maximally flat in the stopband
- Chebyshev II filter has equiripple in the stopband and maximally flat in the passband

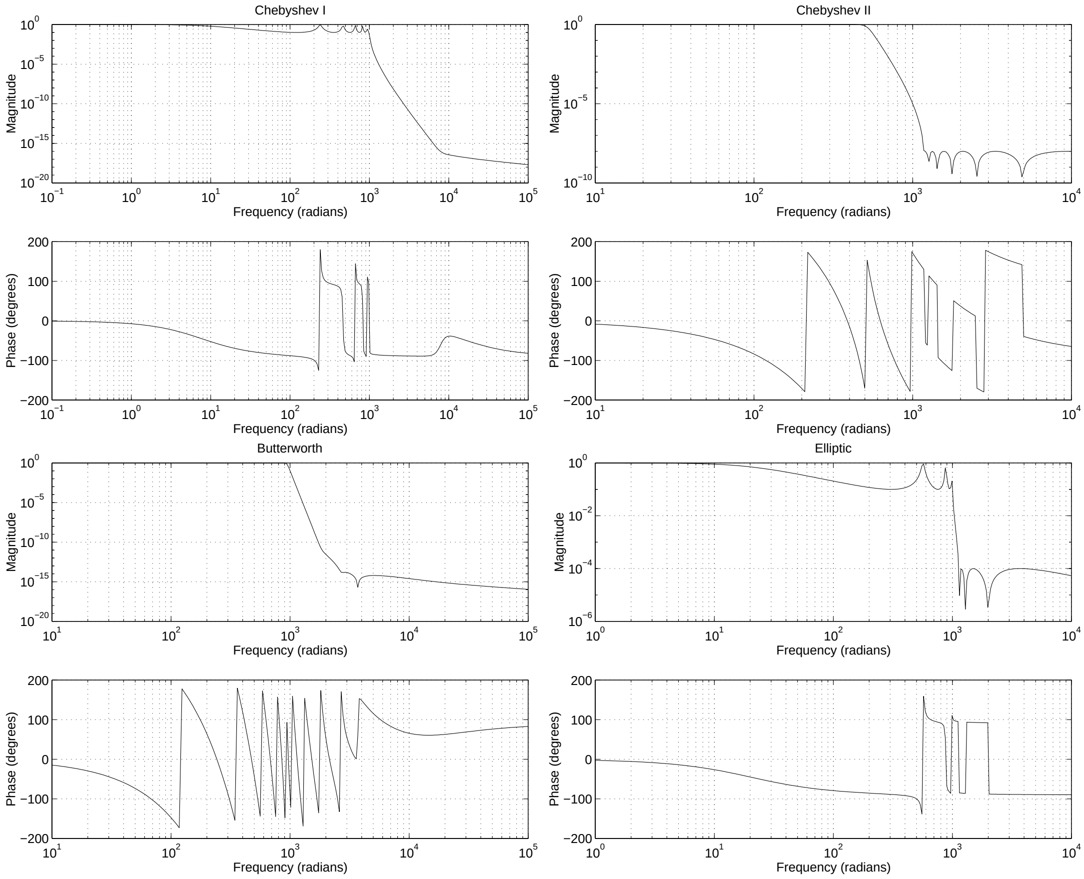

Figure 5.2: Filters.

Part 5.B—Filter Types

Passive filters

The doubly resistive terminated LC-ladder filter prove to be very insensitive to variations in circuit component values.

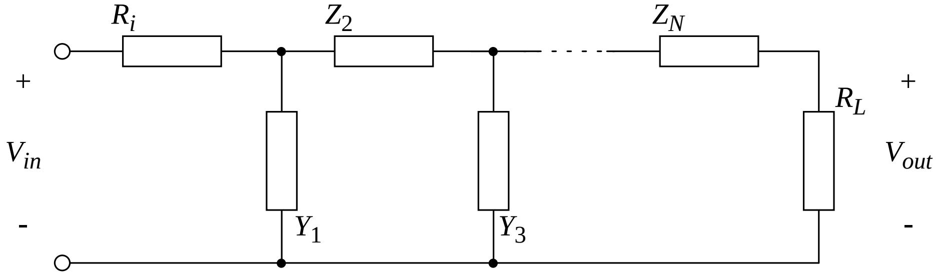

Figure 5.3: Ladder filter structure.

The admittance and impedance value determine the transfer function of the filter. The proper values can be found in tables, but a computer program is preferred.

Passive filters are advantegous when they are to be used in “tough” environments such as front-ends for telephone lines, etc., where the voltage levels can be very high.

However, in this course we focus on the active filters.

Active filters

Basically, when implementing an active filter we try to avoid resistors and inductors, since they cannot (very hard to) be implemented on silicon. To achieve high linearity in resistors we require special layers in our CMOS process. The inductors must be simulated with active components such as the generalized impedance converter (GIC) or through other means.

We use a number of active components, such as the operational amplifier (OP), operational transconductance amplifier (OTA), transconductance amplifier (Gm), in feedback configurations.

There are also different approaches to synthesize the active filters. Among the approaches we point out the use of first- and second-order sections, state-variable, and leapfrog filters.

Continuous-time or discrete-time filters?

Dependent on application we may want to implement discrete-time filters due to higher accuracy. However, some other complexities aries in the design of discrete-time filters. Mostly, we have the switched-capacitor (SC) and switched-current (SI) discrete-time filters.

Since the output signal from a discrete-time filter has images throughout the frequency domain a continuous-time filter is needed to remove these.

Part 5.C—Filter Synthesis

The objective with the following discussions is to find ways to express the filtering functions with a number of **additions, multiplications, and <u>integrators</u>**. The integrator is prefered since it is “simpler” to realize than the deriver and it also has better noise properties due to the $1/s$ slope.

To implement an integrator we also use an amplifier and this also motivates us to find high-accuracy (high-gain) amplifiers during the rest of the course.

“Signal Flow Graphs” (State-variable)

Describing the filter using signal flows or state variables. Assume that the transfer function, $H(s)$, of the filter is given by

$$H(s) = \frac{Y(s)}{X(s)} = \frac{b_0 + b_1 \cdot s + \ldots + b_M \cdot s^M}{a_0 + a_1 \cdot s + \ldots + a_N \cdot s^N}$$

where $Y(s)$ is the output signal and $X(s)$ is the input signal. The equation can be rewritten as

$$H(s) = \frac{Y(s)}{X(s)} = \frac{b_0 \cdot s^{-M} + b_1 \cdot s^{-M+1} + \ldots + b_M}{a_0 \cdot s^{-N} + a_1 \cdot s^{-N+1} + \ldots + a_N} \cdot s^{M-N}$$

The output signal can be written as

$$Y(s) = -\left(\frac{a_0}{a_N}s^{-N} + \frac{a_1}{a_N}s^{-N+1} + \ldots\right)Y(s) + \left(\frac{b_0}{a_N}s^{-M} + \ldots + \frac{b_M}{a_N}\right) \cdot s^{M-N} \cdot X(s)$$

This can further be rewritten as something like

$$Y(s) = ? + s^{-1} \cdot (? + s^{-1} \cdot (? + s^{-1} \cdot (? + \ldots)))$$

This equation may be graphically represented as in Fig. 5.4. We use a number of inte-

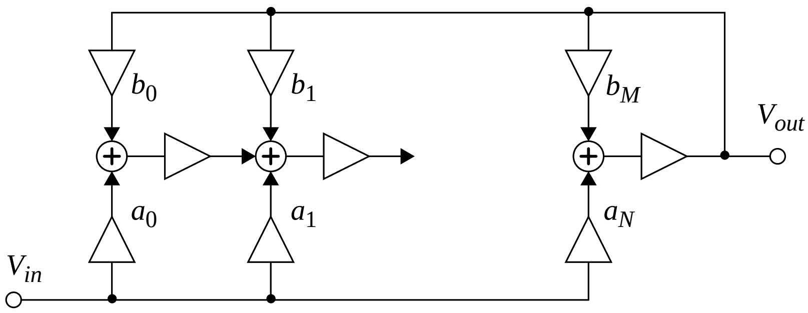

Figure 5.4: Signal flow graph representing the equations of the voltage transfer function.

grators, multipliers, and addition elements. On this behavioral-level, the description holds for both currents and voltages. It is up to the designer to choose which circuit elements that are best suited for implementation.

This approach is simple but the filters become sensitive to variations of circuit components and there might be several buffers needed as well.

Cascaded first- and second-order sections

Another approach is to realize the filter with a number of filtering sections that are cascaded. Hence from the transfer function we extract a number of subexpressions, such that

$$H(s) = H_1(s) \cdot H_2(s) \cdot \ldots \cdot H_M(s)$$

And the idea is sketched in Fig. 5.5 Problems might arise due to input and output

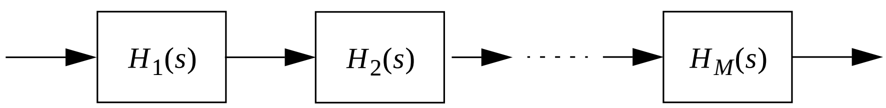

Figure 5.5: Cascaded first- and second-order filter sections.

impedance of the filtering sections and therefore buffers (Gain = 1) must be added between the filtering stages.

Example of a second-order filter section (Bi-quad) is shown in Fig. 5.6. Using different values on the parameters, R, C, n, m, e, f, A we achieve different filtering functions. We may have

$$A = 1,\ \frac{m}{n} = 1,\ e = \frac{1}{m\sigma_p},\ \frac{e}{f} = 4Q^2 = 4\left(\frac{\sigma_p}{2\omega_{0p}}\right)^2$$

Where σ_p is given by $2\sigma_p = \omega_{0p}$, etc.

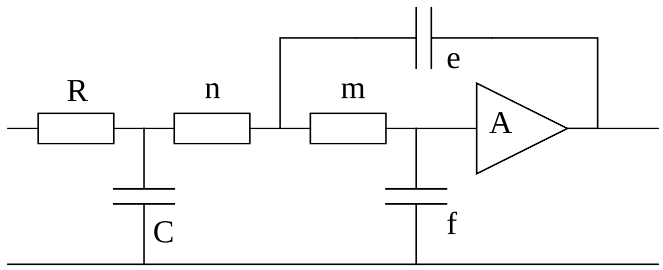

Figure 5.6: Example of a second-order filter section.

This approach is simple but the filters become sensitive to variations of circuit components and there might be several buffers needed as well.

Leapfrog filters

As a good reference filter structure, we point out the ladder filter structure, due to its optimal insensitivity. When we simulate the passive filter with active components, it will inheret the properties of the passive filter, hence the active implementation will definitely not become better than the passive.

For our discussion we consider the ladder filter as illustrated in Fig. 5.7 below.

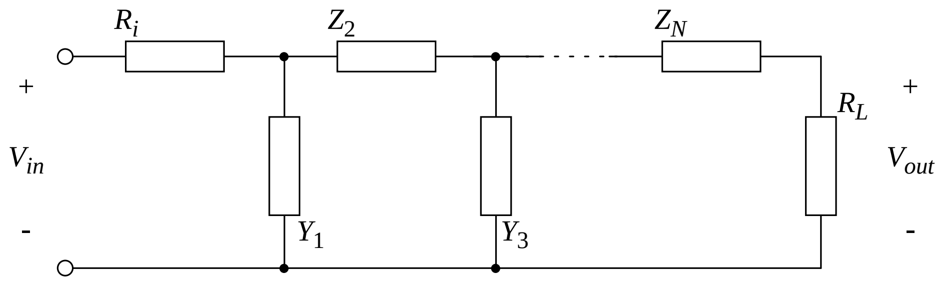

Figure 5.7: Ladder filter structure.

For a leapfrog filter, we investigate the voltages and currents at each node in the ladder structure. In Fig. 5.8 this is examplified and the formulas become

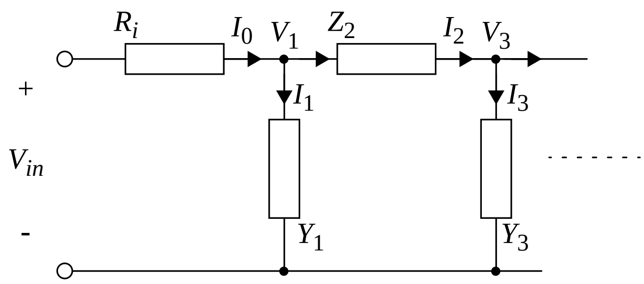

Figure 5.8: Ladder filter with included currents and voltage nodes.

$$V_{in} - R_i \cdot I_0 = V_1,$$
$$V_1 - Z_2 \cdot I_2 = V_3,$$
$$I_2 + Y_1 \cdot V_1 = I_0, \text{ etc.}$$

By using a help resistance R, the equations are rewritten to form voltage-only parameters (states). We have that

$$V_{in} - \frac{R_i}{R} \cdot RI_0 = V_1,$$

$$V_1 - \frac{Z_2}{R} \cdot RI_2 = V_3,$$

$$RI_2 + RY_1 \cdot V_1 = RI_0, \text{ etc.}$$

Using these equation, we may construct a signal flow chart as shown in Fig. 5.9 (a).

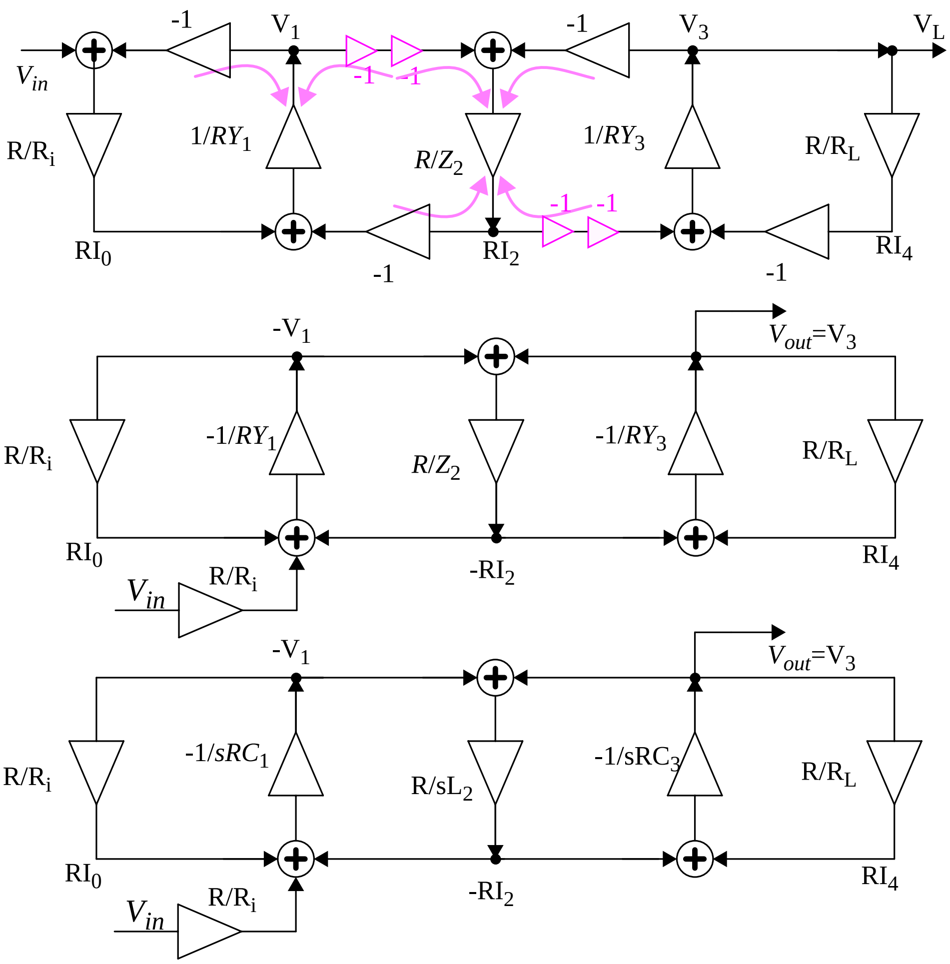

Figure 5.9: Signal flow graph representing the equations for the ladder filter.

We can propagate the inverters through the net and reorder some of the amplifiers, and change the sign of some of the nodes (states), and we get the result in Fig. 5.9(b).

We now see that we have three terms that include addmittances or impedances. If we choose $Z_i = sL_i$ and $Y_i = sC_i$ we will have a number of integrators in the flow instead as shown in Fig. 5.9(c).

The corresponding ladder structure is shown in Fig. 5.9 (b).

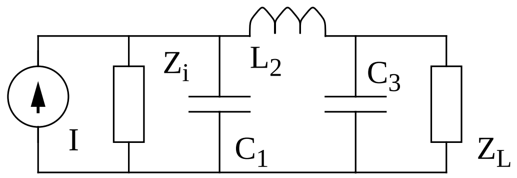

Figure 5.10: Using inductors and capacitors give integrators in the signal flow graph.

Part 5.D—Circuit realization of filters

We can realize the integrators in several different ways. The first approach and hopefully well-known is using an operational amplifier (OP). The second approach can be to use a transconductance-C (Gm-C) circuit. These two types are shown in Fig. 5.11.

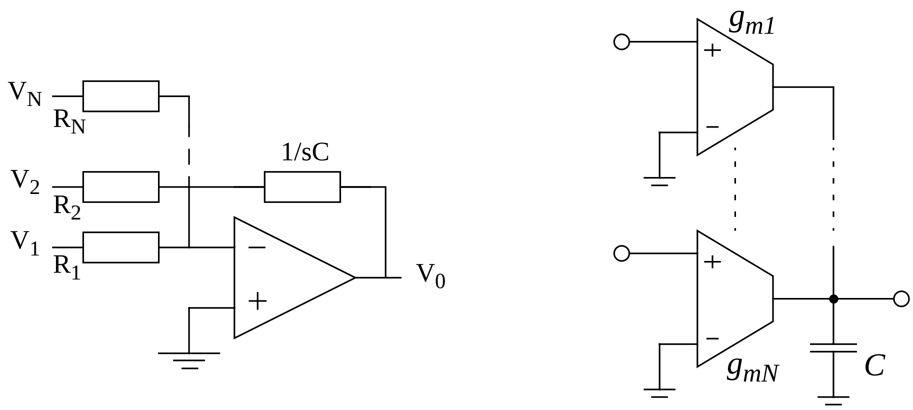

Figure 5.11: Summing integrators using (a) OP and (b) Gm.

We find the output voltages as

$$V_{out} = \frac{-1}{sR_1C} \cdot V_1 + \ldots + \frac{-1}{sR_NC} \cdot V_N \text{ for the OP and}$$

$$V_{out} = \frac{g_{m1}}{sC} \cdot V_1 + \ldots + \frac{g_{mN}}{sC} \cdot V_N \text{ for the Gm-C.}$$

Summations and gain factors can be realized with similar topologies as shown in Fig. 5.12 We find the output voltages as

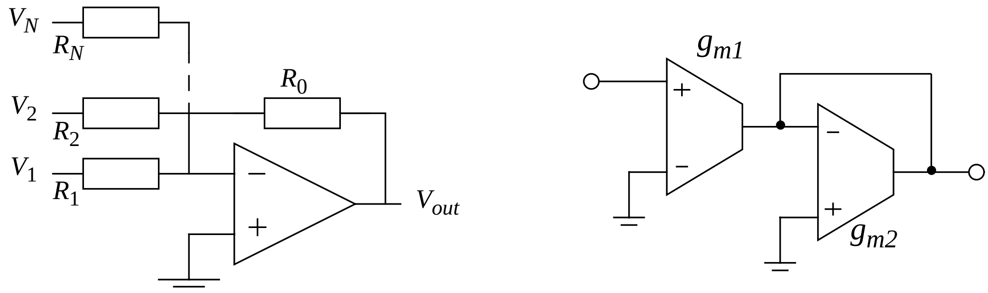

Figure 5.12: Summing amplifiers using (a) OP and (b) Gm.

$$V_{out} = \frac{-R_0}{R_1} \cdot V_1 + \ldots + \frac{-R_0}{R_N} \cdot V_N \text{ for the OP and } V_{out} = \frac{g_{m1}}{g_{m2}} \cdot V_{in} \text{ for the Gm.}$$

OPamp realization of leapfrog filter

In Fig. 5.13 we show the corresponding leapfrog filter implemented with operational amplifier.

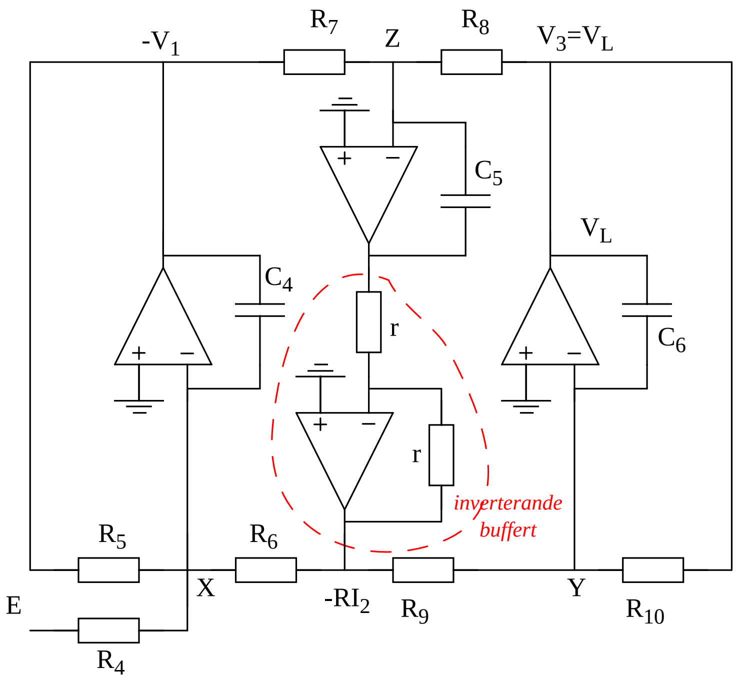

Figure 5.13: OPamp realization of leapfrog filter.

To find the correct values we compare the signal flowgraph with the OP leapfrog implementation. Compare the signal path from V_i to V_j $([V_j]_{V_i})$.

Leapfrog	Signal flow graph	Result
$[-V_1]_E = \frac{-1}{sC_4} \cdot \frac{E}{R_4}$	$[-V_1]_E = \frac{R}{R_i} \cdot \frac{-E}{sRC_1} = \frac{-E}{sR_iC_1}$	$C_4R_4 = C_1R_i$
$[-V_1]_{-V_1} = -\frac{1}{sC_4} \cdot \frac{V_1}{R_5}$	$[-V_1]_{-V_1} = \frac{R}{R_i} \cdot \frac{--V_1}{sRC_1} = \frac{V_1}{sR_iC_1}$	$C_4R_5 = C_1R_i$
$[-V_1]_{-RI_2} = -\frac{1}{sC_4}\frac{-RI_2}{R_6}$	$[-V_1]_{-RI_2} = -\frac{1}{sRC_1}(-RI_2)$	$C_4R_6 = C_1R$
$[-RI_2]_{-V_1} = -\frac{(-1)}{sC_5} \cdot \frac{-V_1}{R_7}$	$[-RI_2]_{-V_1} = \frac{R}{sL_2}(-V_1)$	$C_5R_7 = L_2/R$
$[-RI_2]_{V_3} = -\frac{(-1)}{sC_5} \cdot \frac{V_3}{R_8}$	$[-RI_2]_{V_3} = \frac{R}{sL_2}V_3$	$C_5R_8 = L_2/R$
$[V_3]_{-RI_2} = -\frac{1}{sC_6}\frac{-RI_2}{R_9}$	$[V_3]_{-RI_2} = -\frac{1}{sRC_3}(-RI_2)$	$C_6R_9 = C_3R$
$[V_3]_{V_3} = -\frac{1}{sC_6}\frac{V_3}{R_{10}}$	$[V_3]_{V_3} = \frac{R}{R_L}V_3\frac{-1}{sRC_3} = \frac{-V_3}{sC_3R_L}$	$C_6R_{10} = C_3R_L$

There are more equations than variables and the best start is to set all capacitors equally large, hence

$$C_4 = C_5 = C_6 = C_0$$

We can then derive the corresponding resistance values. The resistance values on the inverting buffer, r can be chosen so that it is in the same order of magnitude as the other filter resistances. Also choose the values so that you achieve symmetry. And finally you should choose the values to be as equal as possible (hence low variation in parameter values).

Finally, we still may have to scale the filter and then some of the component values will change.

Gm-C realization of leapfrog filter

Using the same signal flow graph we can easily find the Gm-C realization of the filter as illustrated in Fig. 5.14.

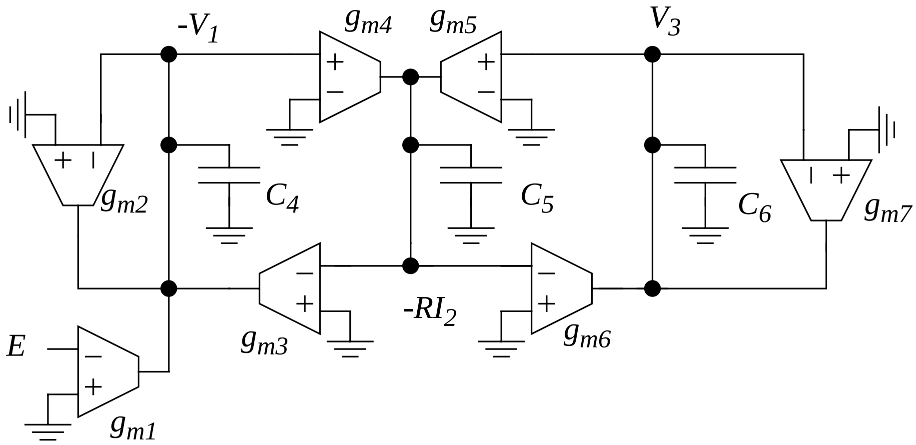

Figure 5.14: Gm-C implementation of the leapfrog filter.

Once again by identifying the signal flow graphs we find requirements on the component values. We choose all capacitors to be equally large and then we find the sizes of the transconductances and let them have as small variation as possible. The symmetry should also be utilized.

Part 5.E—Cauer filters

When dealing with cauer filters we have to be careful with capacitors in parallel with inductors. A third order Cauer filter is shown in Fig. 5.15.

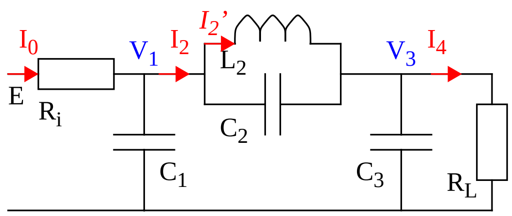

Figure 5.15: Cauer (elliptic) filters contain capacitors in the inductive branches as well.

Introduce current and voltages and find the equations. Using a normalized resistance, R.

$$I_0 = \frac{E - V_1}{R_i} \qquad RI_0 = \frac{R}{R_i}(E - V_1)$$

$$V_1 = \frac{1}{sC_1}(I_0 - I_2) \qquad V_1 = \frac{1}{sRC_1}(RI_0 - RI_2)$$

$$I_2 = \frac{1}{L_2 \| C_2}(V_1 - V_3) \qquad RI_2 = \frac{R}{sL_2/(1 + s^2 L_2 C_2)}(V_1 - V_3)$$

$$V_3 = \frac{1}{sC_3}(I_2 - I_4) \qquad V_3 = \frac{1}{sRC_3}(RI_2 - RI_4)$$

$$I_4 = \frac{V_3}{R_L} \qquad RI_4 = \frac{R}{R_L}V_3$$

However, it shows that we have a double zero for some of the equations due to the capacitor in parallel with the inductor. To overcome this problem we introduce a help current through the inductor, I_2'. we modify the equations according to:

$$RI_2 = RI_2' + sRC_2(V_1 - V_3) \text{ and } RI_2' = \frac{R}{sL_2}(V_1 - V_3)$$

The expression RI_2 is eliminated in both equation for V_1 and V_3:

$$V_1 = \frac{1}{sRC_1}(RI_0 - RI_2' - sRC_2(V_1 - V_3)) \Rightarrow$$

$$V_1 = \frac{1}{sR(C_1 + C_2)}(RI_0 - RI_2') + \frac{C_2}{C_1 + C_2}V_3$$

In the same way, we have

$$V_3 = \frac{1}{sR(C_2 + C_3)}(RI_2' - RI_4) + \frac{C_2}{C_2 + C_3}V_1$$

This will simulate two voltage-dependent voltage sources as illustrated in Fig. 5.16.

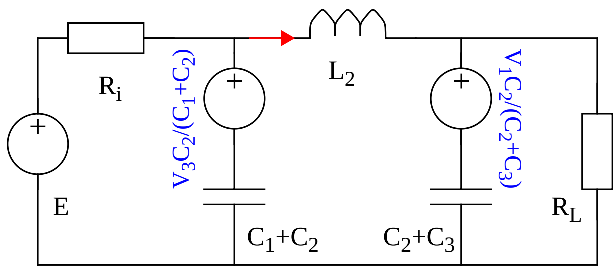

Figure 5.16: Cauer filter with voltage-controlled voltage sources.

Basically, the influence of C_2 is now “moved” to the C_1 and C_3 branches. Using the equations and the figure above, we can form the new signal flow graph as shown in Fig. 5.17 where $\alpha_j = C_j + C_2$.

The corresponding OPamp realization is found in Fig. 5.18. By comparing the signal paths through the two representations we can find the relations between circuit components and specified values.

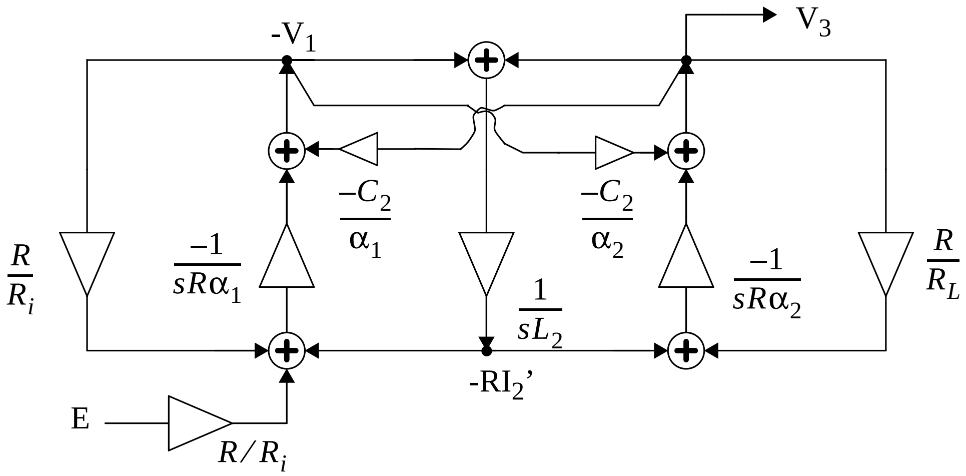

Figure 5.17: Cauer filter signal flow graph.

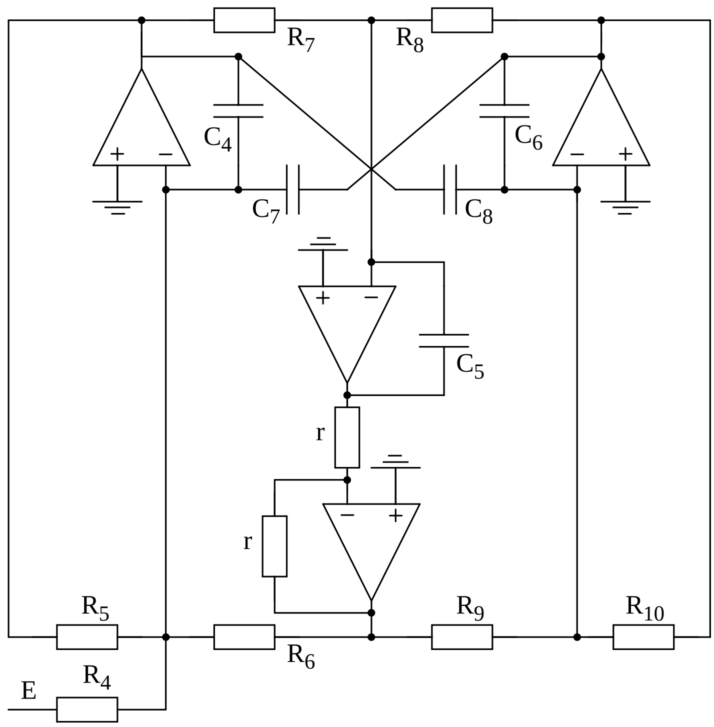

Figure 5.18: Ladder filter with included currents and voltage nodes.

We have that:

OP leapfrog	Signal flow graph	Result
$[-V_1]_E = -\frac{1}{sC_4}\cdot\frac{E}{R_4}$	$[-V_1]_E = \frac{R}{R_i}\cdot\frac{-E}{sR\alpha_1} = \frac{-E}{sR_i\alpha_1}$	$C_4R_4 = \alpha_1R_i$
$[-V_1]_{-V_1} = -\frac{1}{sC_4}\cdot\frac{V_1}{R_5}$	$[-V_1]_{-V_1} = \frac{R}{R_i}\cdot\frac{-V_1}{sR\alpha_1} = \frac{-V_1}{sR_i\alpha_1}$	$C_4R_5 = \alpha_1R_i$
$[-V_1]_{-RI_2'} = -\frac{1}{sC_4}\frac{-RI_2}{R_6}$	$[-V_1]_{-RI_2} = -\frac{1}{sR\alpha_1}(-RI_2)$	$C_4R_6 = \alpha_1R$

$$[-V_1]_{-V_3} = -\frac{C_7}{C_4}V_3 \qquad [-V_1]_{-V_3} = -\frac{C_2}{\alpha_1}V_3 \qquad C_4C_2 = C_7\alpha_1$$

$$[-RI_2]_{-V_1} = -\frac{(-1)}{sC_5}\cdot\frac{-V_1}{R_7} \qquad [-RI_2]_{-V_1} = \frac{R}{sL_2}(-V_1) \qquad C_5R_7 = L_2/R$$

$$[-RI_2]_{V_3} = -\frac{(-1)}{sC_5}\cdot\frac{V_3}{R_8} \qquad [-RI_2]_{V_3} = \frac{R}{sL_2}V_3 \qquad C_5R_8 = L_2/R$$

$$[V_3]_{-RI_2} = -\frac{1}{sC_6}\frac{-RI_2}{R_9} \qquad [V_3]_{-RI_2} = \frac{-1}{sR\alpha_2}(-RI_2) \qquad C_6R_9 = \alpha_2R$$

$$[V_3]_{V_3} = -\frac{1}{sC_6}\frac{V_3}{R_{10}} \qquad [V_3]_{V_3} = \frac{R}{R_L}V_3\frac{-1}{sR\alpha_2} = \frac{-V_3}{s\alpha_2R_L} \qquad C_6R_{10} = \alpha_2R_L$$

$$[V_3]_{-V_1} = -\frac{C_8}{C_6}(-V_1) \qquad [V_3]_{-V_1} = -\frac{C_2}{\alpha_2}(-V_1) \qquad C_2C_6 = C_8\alpha_2$$

Where $\alpha_1 = C_1 + C_2$ and $\alpha_2 = C_2 + C_3$.

Gm-C realization of Cauer leapfrog filter

Using the same signal flow graph we can easily find the Gm-C realization of the filter as illustrated in Fig. 5.14. However, there is now a floating capacitor in the filter which

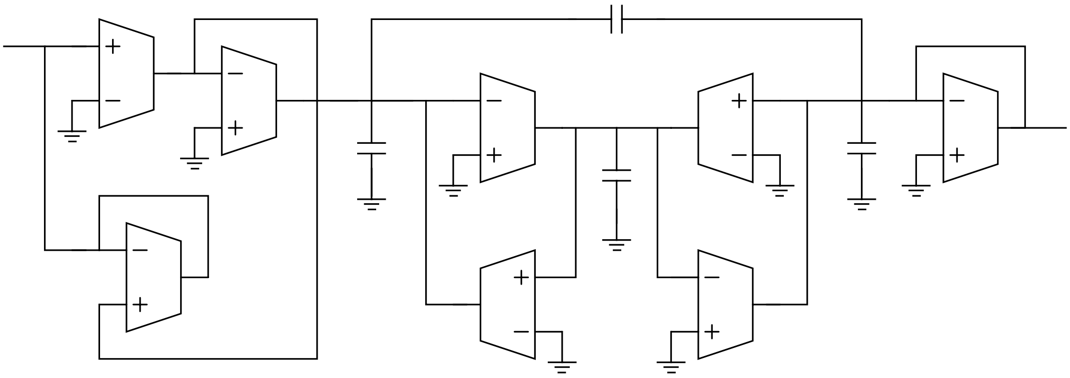

Figure 5.19: Gm-C implementation of the Cauer leapfrog filter.

should be avoided. One approach to do this is to rewrite the voltage controlled voltage sources as voltage controlled current sources (Norton theorem) as shown in Fig. 5.20.

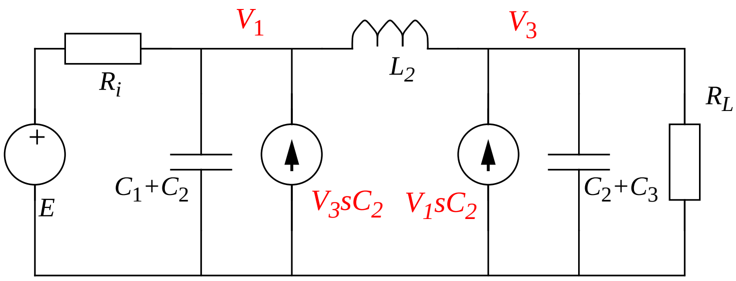

Figure 5.20: Modification of the Cauer reference ladder filter to suit a Gm-C realization.

We have to realize the current sources which can be done by using OPs together with the Gm-C element but this is not part of the course.

Once again by identifying the signal flow graphs we find requirements on the component values. We choose all capacitors to be equally large and then we find the sizes of the transconductances and let them have as small variation as possible. The symmetry should also be utilized.

Part 5.F—Discrete-time filters

We will briefly discuss some different approaches to realize discrete-time filters. Later on when we discuss SC filter we will come back to this.

Cascaded first- and second-order sections

The first filter implementation is straight-forward where we use a number of cascaded filter sections similar to the continuous-time case.

Signal flow graph or state variable

The second approach is to consider the transfer function of a discrete-time filter, e.g.

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 \cdot z^{-1} + \ldots + b_M \cdot z^{-M}}{a_0 + a_1 \cdot z^{-1} + \ldots + a_N \cdot z^{-N}}$$

From this representation, we can generate a signal flow chart, similar to the one for the continuous-time filters. However, notice that a discrete-time integrator (accumulator) has a transfer function as

$$I(z) = \frac{z^{-1}}{1 - z^{-1}}$$

and there might be some tricks needed to find subexpressions in the transfer function containing this accumulator function.

Transformation from continuous-time specification

We will however discuss the lossless discrete integrator (LDI) and bilinear transforms. We use the continuous-time filter as reference and then we transform into a discrete-time representation.

The LDI transform is given by

$$s = s_0 \cdot \frac{z - 1}{z^{1/2}}$$

where s_0 is a normalizing constant. Although the LDI transform is approximate, it is mostly preferred over the bilinear transform. The bilinear transform is given by

$$s = \gamma \cdot \frac{z - 1}{z + 1}$$

where γ is a normalizing constant. The bilinear transform i “exact” but gives a more complex circuit implementation.

LDI manipulation of continuous-time reference filter

First one has to be careful with what is actually specified in a discrete-time filter. Mostly, the wanted “analog” output specification is given. This implies that we have to make a multi-stage transformation. Consider the case in Fig. 5.21.

We first consider the reconstructed specification. It will give us the required parameters for the passband

$$A_{min} \text{ and } f_p$$

and the parameters for the stopband

$$A_{max} \text{ and } f_s$$

We will also get the sample frequency

$$f_{sample} = 1/T$$

We first consider the Poission formula

$$X(e^{j\omega t}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X\left(j\omega - j\frac{2\pi}{T}k\right)$$

Hence what are the normalized angular frequencies, $\omega_p T$ and $\omega_s T$, for the discrete-time case. This will give us the sampled filter specification.

Then we apply the LDI transform inversely to find the proper reference filter specification. Hence we want to find the ω_p and ω_s used in a reference filter so that the sampled filter will give us the true result inspite of the errors that the LDI transform introduces.

Assume that we find a reference filter and we de-normalize the values, etc.

We find the signal flow graph and then we should apply the LDI transform to this flow graph. Consider for example the Cauer filter shown in Fig. 5.17. Replace all continuous-time integrators corresponding to the LDI transform:

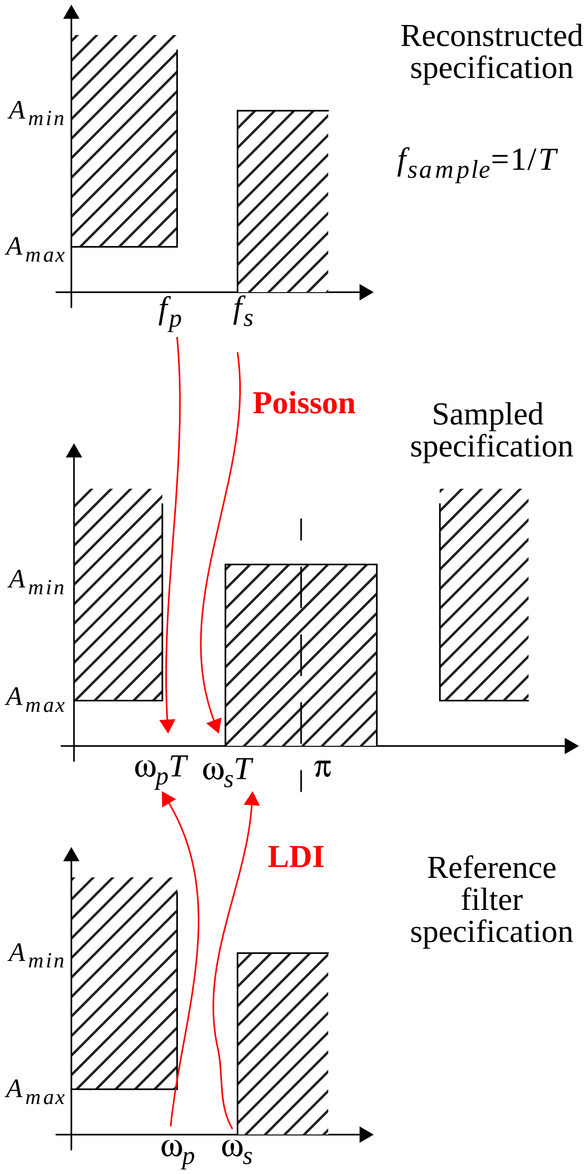

Figure 5.21: Modification of the Cauer reference ladder filter to suit a Gm-C realization.

$$s = s_0 \cdot \frac{z-1}{z^{1/2}}$$

This will give the result shown in Fig. 5.25(a). We must now eliminate all the $z^{-1/2}$ factors. They can be "moved around" in the graph and we end up with the result shown in Fig. 5.22(b).

We now have a problem with the frequency-dependent outer branches:

$$\frac{R}{R_L} \cdot z^{-1/2}$$

which also has to be eliminated.

It can be found that

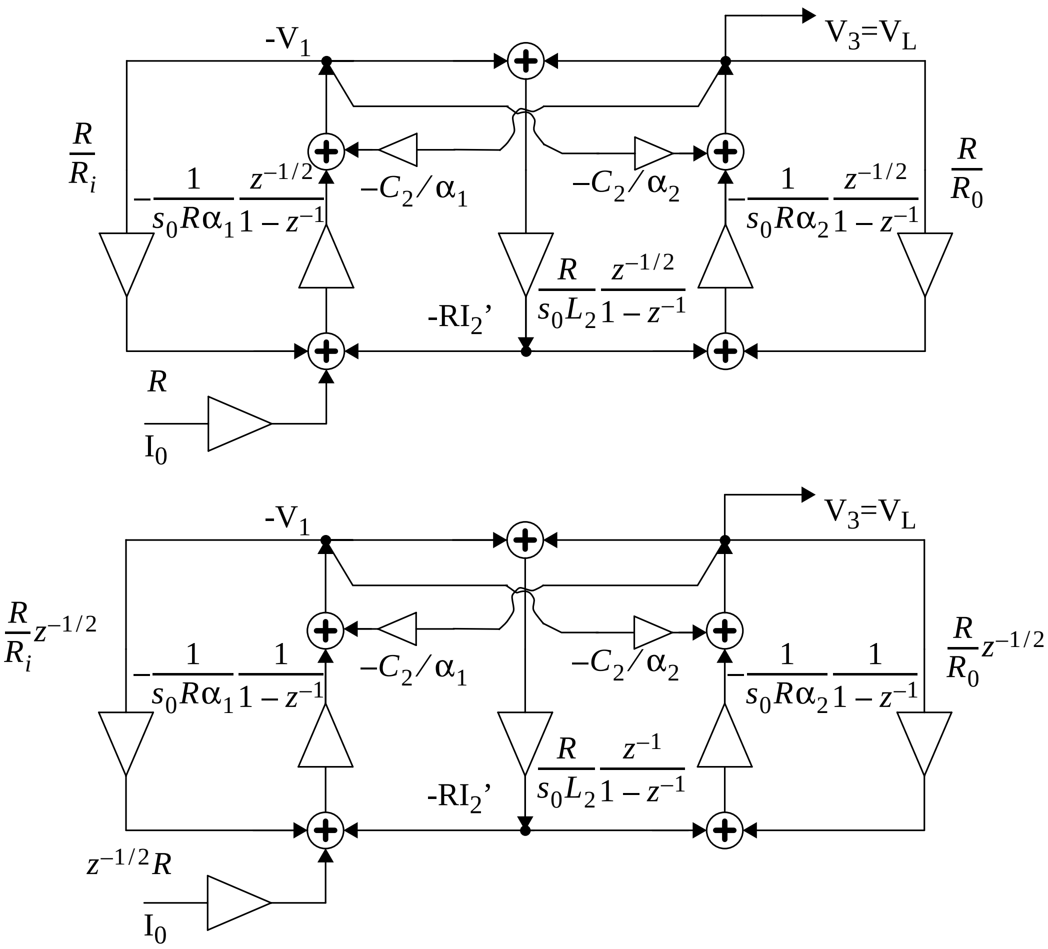

Figure 5.22: The Cauer reference ladder filter where integrators are replaced with discrete-time accumulators.

$$z^{-1/2} = -j\frac{\omega}{2s_0} + \sqrt{1-\left(\frac{\omega}{2s_0}\right)^2}$$

We can approximate R_L with $R_L \cdot z^{-1/2}$ and the resistance can be seen as a frequency dependent resistance in parallel with a capacitor as illustrated in Fig. 5.23:

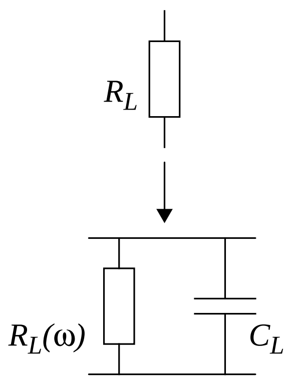

Figure 5.23: Approximation of frequency-dependent resistance.

$$R_L \rightarrow R_L(\omega) \parallel C_L$$

This will give a modification of the circuit values as

$$C_1' = C_1 - C_i = C_1 - \frac{1}{2s_0R_i} = C_1 - \frac{\sin(\Omega_0 T/2)}{\omega_0 R_i}$$

$$C_3' = C_3 - C_L = C_3 - \frac{1}{2s_0R_L} = C_3 - \frac{\sin(\Omega_0 T/2)}{\omega_0 R_L}$$

α_1 and α_2 are changed into α_1' and α_2', respectively. Finally we get the signal flow graph shown in Fig. 5.24.

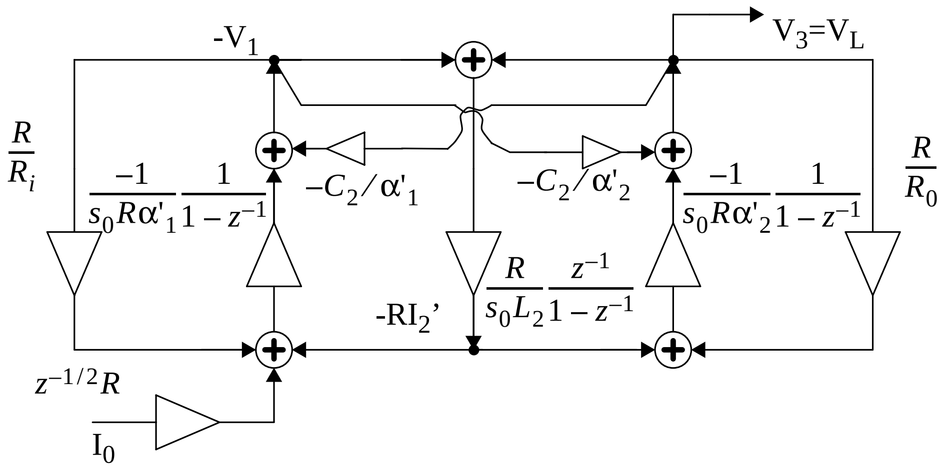

Figure 5.24: The Cauer reference ladder filter where integrators are replaced with discrete-time accumulators.

Sinc-weighting of signal spectrum

Due to the discrete-time properties (sample&hold at the output) the signal spectrum will also become sinc weighted (as illustrated in Fig. 5.25). However, this can mostly

Figure 5.25: Resulting sinc-weighting of the output spectrum.

be compensated for by choosing the filter coefficients in a smart way. The images in the frequency-domain have to be removed with a continuous-time filter.

To not have to care about the correction of the coefficents, one can choose to have a much higher sampling frequency than signal frequency. However, this will give rise to other problems.

Part 5.G—Filter scaling

A filter should be scaled for optimum performance.

- The signals are usually limited by the supply voltages, hence the signal swing $< V_{DD} - V_{SS}$
- Small signal swing gives a low SNR

Change the signal swing in "important" nodes to have

$$\max \left| H_i(j\omega) \right| = \max \frac{\left| X_i(j\omega) \right|}{V_{in}(j\omega)} = 1 \text{ (max over all frequencies).}$$

Principle of scaling

Multiply all input signals to a sub-net by a factor k_i and divide all output signal with the same factor. The principle above is based on that if we do not change the loop gains in the signal flow graph the poles of the transfer function are fixed.

In Fig. 5.26 we show an example of scaling of a third-order leapfrog filter.

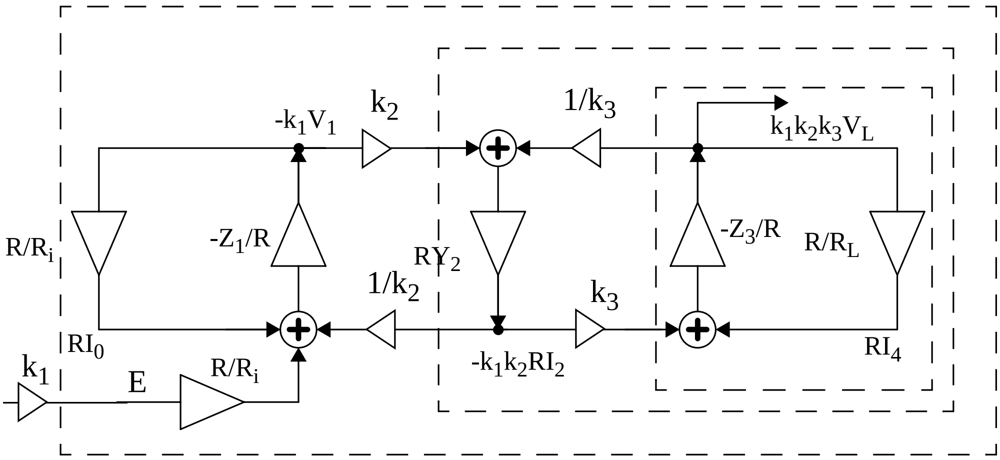

Figure 5.26: Example on scaling of a third-order filter.