TSEK37
Analog CMOS Integrated Circuits

Examination (TEN1)

Time:	16 January 2016 at 14.00 - 18.00
Place:	G32
Responsible teacher:	Martin Nielsen-Lönn, ISY, 070-361 52 44 (martin.nielsen.lonn@liu.se)
	Will visit exam location at 15 and 17.
Number of tasks:	6
Number of pages:	7
Allowed aids:	Calculator, dictionary
Total points:	20
Notes:	Remember to indicate the steps taken when solving problems.
	Please start each new problem at the top of a page!
	Only use one side of each paper!
Exam presentation:	22 Januari 2016 at 12:00-13:00 in 3D:535, B-building

Grade	**Points**
U	<8
3	8 - <12
4	12 - <16
5	16 - 20

Questions

1) Consider the circuit shown in Fig. 1.

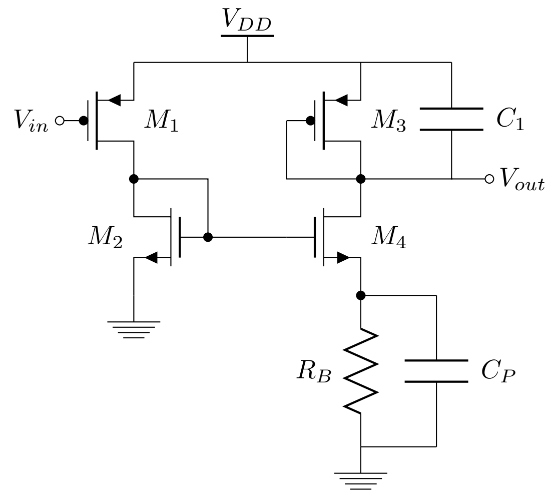


Figure 1: Schematic of a circuit.

(a) Draw the equivalent small-signal model and derive the expression for the transfer function and from that transfer function identify the DC voltage gain, pole(s) and zero(s). Neglect channel-length modulation and body effect ($\lambda = 0$, $\gamma = 0$). Ignore all parasitic capacitances except C_p and C_1. (3)

(b) For the circuit in Fig. 1, let the unity-gain frequency $\omega_{ug} = 20$ krad/s, the dominant pole $\omega_{p1} = 1$ krad/s and the phase margin PM = 55°. Determine the value of the non-dominant pole ω_{p2}. Ignore the effect of the zero(s) on the PM. (1)

---

2) Consider the fully differential amplifier shown in Fig. 2. The amplifier is completely symmetric. Neglect body effect ($\gamma = 0$). However, channel-length modulation must be considered ($\lambda \neq 0$). The parasitic capacitance C_{gs} of the MOSFETs at nodes X, Y and the load capacitance C_L must be considered. Ignore other parasitic capacitances. Assume that C_{gs} of $M_{4a,4b} = B \times C_{gs}$ of $M_{2a,2b}$, $g_{m4a,4b} = B \times g_{m2a,2b}$ and $g_m \gg g_{ds}$. B is an adjustable parameter.

(a) Draw the small-signal model of the amplifier shown in Fig. 2 and derive the transfer function. (2½)

(b) Derive the expressions for the DC gain, the two pole frequencies and the unity-gain frequency. It is assumed that the dominant pole is formed by the load capacitor C_L. (1)

(c) If the factor B is increased, how will the non-dominant pole and unity-gain frequency change? It is assumed that the dominant pole is formed by the load capacitor C_L. (½)

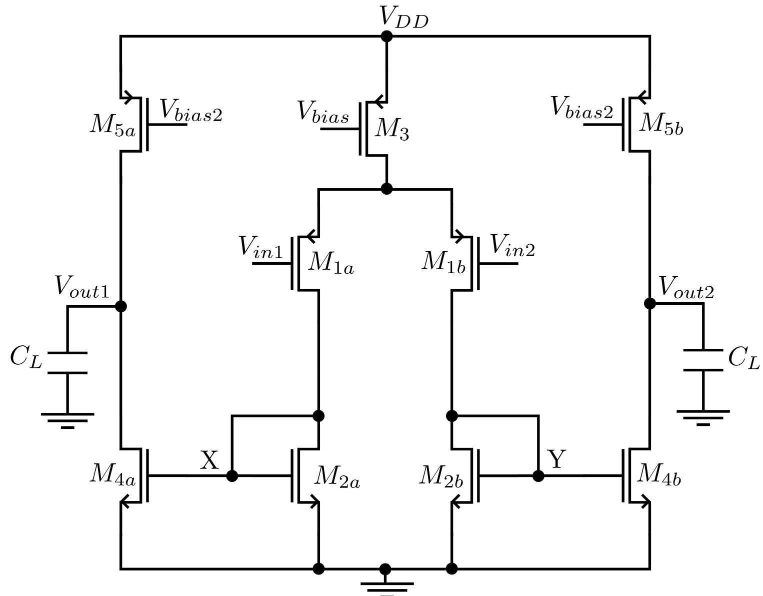


Figure 2: A fully differential amplifier.

---

3) In Fig. 3 a wide-swing cascode current mirror is shown. The aim here is to copy the reference current I_1 to I_2. Assume that all transistors are in saturation and have the same threshold voltage. Neglect body effect and channel-length modulation. The transistor M_2 is designed to be at the edge of the saturation region to have the maximum voltage swing at the output node. What should be the value of n such that $I_1 = I_2$? (3)

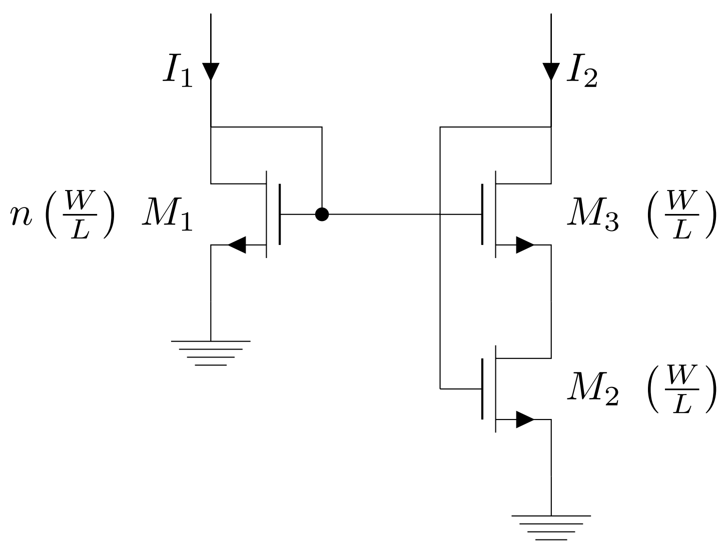


Figure 3: A wide-swing cascode current mirror.

---

4) Figure 4 shows a lossless interconnect circuit. At $t = 0$ the voltage step V_0 goes high and propagates a pulse through the circuit. Given the values below, calculate the voltage at nodes A, B and C at $t = 7$ ns. t_{d,Z_x} is the delay through transmission line Z_x. Assume that the inverters have a infinite (high) input impedance. (3)

$R_0 = 150\ \Omega$, $Z_0 = 150\ \Omega$, $Z_1 = 75\ \Omega$, $Z_2 = 50\ \Omega$, $t_{d,Z_0} = 2$ ns, $t_{d,Z_1} = 3$ ns and $t_{d,Z_2} = 4$ ns.

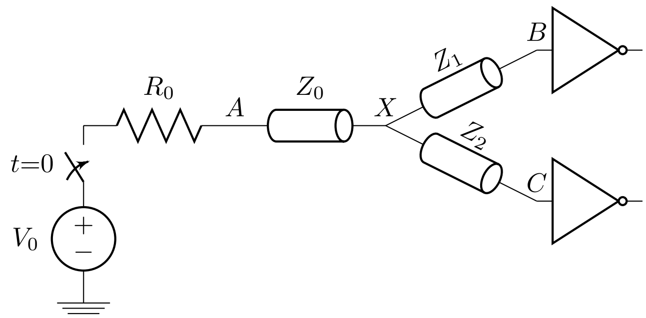


Figure 4: Transmission line circuit.

---

5) Assume that an inverter stage can be approximated as a first order circuit with a single pole at ω_0 and a stage DC-gain of $-A$.

(a) What are the possible number of stages required to construct a ring oscillator and what is the minimum number assuming each stage is made using a single-ended inverter? How would this change if differential inverters are used instead and why? (1)

(b) Draw such a ring oscillator with a minimum amount of stages. Derive the open-loop transfer function for this ring oscillator. (1)

(c) Calculate the minimum required voltage gain per stage in order to have oscillation and how this gain requirement changes with the number of stage, N. (1½)

(d) Assume that the ring oscillator has settled into its steady state which occurs when the output signals starts to swing from roughly 0 to roughly V_{DD}. At this point the propagation delay of each stage is $t_{p,s}$. What is the output frequency if we have N number of stages? (½)

---

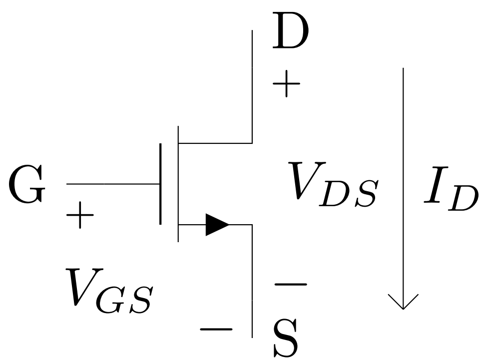
6) A PLL type I contains a low-pass filter, phase detector and VCO.

(a) Draw the block diagram of the PLL, derive the total transfer function as $H(s) = \frac{\theta_{out}(s)}{\theta_{in}(s)}$ and show that θ_{out} is equal to θ_{in} when θ_{in} is constant. (1½)

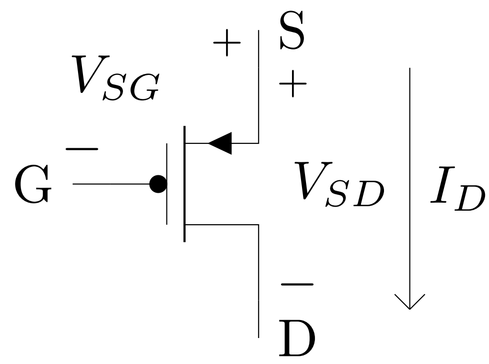
$$H_{PD}(s) = K_{PD}[\theta_{in}(s) - \theta_{out}(s)]$$
$$H_{LP}(s) = \frac{1}{1 + \frac{s}{\omega_{p1}}}$$
$$H_{VCO}(s) = \frac{K_{VCO}}{s}$$

(b) A PLL can be used to multiply a frequency. Assume that the PLL is supposed to generate a frequency of 1 GHz from a 250 MHz clock. Modify and insert additional needed blocks in the block diagram to implement this behaviour. (½)

Transistor equations



(a) NMOS



(b) PMOS

NMOS

Cutoff $I_D = 0$ $(V_{GS} < V_{TN})$

Linear mode

$$I_D = \mu_n C_{ox} \frac{W}{L} \left((V_{GS} - V_{TN}) V_{DS} - \frac{V_{DS}^2}{2} \right) \quad (V_{GS} > V_{TN}) \text{ and } (V_{DS} < V_{GS} - V_{TN})$$

Saturation mode

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TN})^2 (1 + \lambda V_{DS}) \quad (V_{GS} > V_{TN}) \text{ and } (V_{DS} > V_{GS} - V_{TN})$$

PMOS

Cutoff $I_D = 0$ $(V_{SG} < |V_{TP}|)$

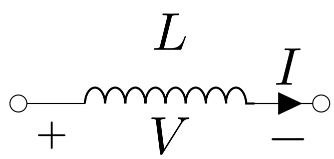
Linear mode

$$I_D = \mu_p C_{ox} \frac{W}{L} \left((V_{SG} - |V_{TP}|) V_{SD} - \frac{V_{SD}^2}{2} \right) \quad (V_{SG} > |V_{TP}|) \text{ and } (V_{SD} < V_{SG} - |V_{TP}|)$$

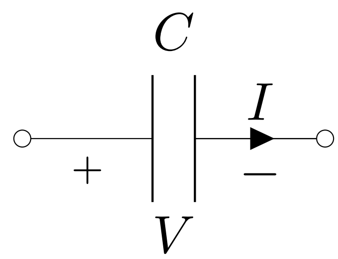
Saturation mode

$$I_D = \frac{1}{2} \mu_p C_{ox} \frac{W}{L} (V_{SG} - |V_{TP}|)^2 (1 + \lambda V_{SD}) \quad (V_{SG} > |V_{TP}|) \text{ and } (V_{SD} > V_{SG} - |V_{TP}|)$$

Transmission line equations



(a) Inductor



(b) Capacitor

Complex characteristic impedance

$$Z_c = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

Characteristic impedance for lossless TL

$$Z_0 = \sqrt{\frac{L}{C}}$$

Inductance voltage-current relation

$$V = L\frac{dI}{dt}$$

Capacitance voltage-current relation

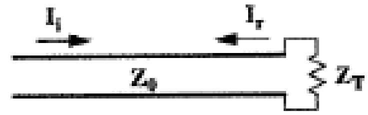
$$I = C\frac{dV}{dt}$$

Mutual inductance

$$V_{mn} = L_{mn}\frac{dI_n}{dt} \text{ where } m \neq n$$

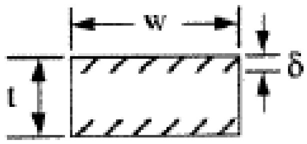
Telegrapher's Equation

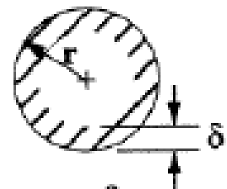
Reflection Coefficient $k_r = \frac{V_r}{V_i} = \frac{I_r}{I_i} = \frac{Z_T - Z_0}{Z_T + Z_0}$



Skin Effect

Skin Depth $\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$

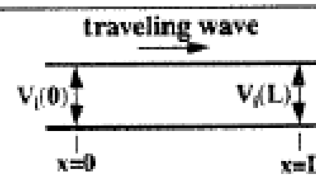




Skin Depth Frequency $f_s = \frac{\rho}{\pi\mu(t/2)^2}$ $\qquad f_s = \frac{\rho}{\pi\mu r^2}$

Skin Depth Resistance $R(f) = \frac{\sqrt{\pi f \mu \rho}}{2w} = R_{DC}\left(\frac{f}{f_s}\right)^{1/2}$ $\qquad R(f) = \frac{\sqrt{f\mu\rho/\pi}}{2r} = \frac{R_{DC}}{2}\left(\frac{f}{f_s}\right)^{1/2}$

Attenuation in Lossy Line

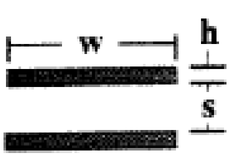


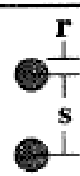
Attenuation $\frac{V_i(L)}{V_i(0)} = \exp[-(\alpha_R + \alpha_D)L] = \exp\left[-\left(\frac{R}{2Z_0} + \frac{GZ_0}{2}\right)L\right]$

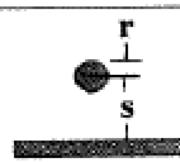
Conductor Loss $\alpha_R(f) = \frac{R_{DC}}{4Z_0}\left(\frac{f}{f_s}\right)^{1/2}$ **(Round)** $\qquad \alpha_R(f) = \frac{R_{DC}}{2Z_0}\left(\frac{f}{f_s}\right)^{1/2}$ **(Strip)**

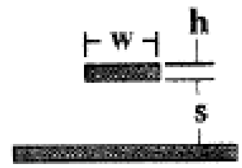
Dielectric Loss (Homogeneous) $\alpha_D(f) = \frac{\pi\sqrt{\varepsilon_r}\tan\delta}{c} f$ $\qquad$ **Dielectric Loss Tangent** $\tan\delta = \frac{G}{\omega C} = \frac{\sigma_{Diel}}{\omega\varepsilon_r}$

R,C,Z_0 for Various Geometries (Homogeneous Dielectric, $L = \varepsilon\mu/C$)









$R_{DC} = 2\frac{\rho}{wh}$	$R_{DC} = \frac{\rho}{\pi r_1^2} + \frac{\rho}{\pi(r_3^2 - r_2^2)}$	$R_{DC} = \frac{2\rho}{\pi r^2}$	$R_{DC} = \frac{\rho}{\pi r^2}$	$R_{DC} = \frac{\rho}{wh}$
$C = \frac{\varepsilon w}{s}$	$C = \frac{2\pi\varepsilon}{\log(r_2/r_1)}$	$C = \frac{\pi\varepsilon}{\log(s/r)}$	$C = \frac{\pi\varepsilon}{\log(2s/r)}$	$C = \frac{\varepsilon w}{s} + \frac{2\pi\varepsilon}{\log(s/w)}$
$Z_0 = \sqrt{\frac{\mu}{\varepsilon}}\frac{s}{W}$	$Z_0 = \sqrt{\frac{\mu}{\varepsilon}}\frac{\log(r_2/r_1)}{2\pi}$	$Z_0 = \sqrt{\frac{\mu}{\varepsilon}}\frac{\log(s/r)}{\pi}$	$Z_0 = \sqrt{\frac{\mu}{\varepsilon}}\frac{\log(2s/r)}{\pi}$	$Z_0 = \frac{\sqrt{\varepsilon\mu}}{C}$