Behzad Mesgarzadeh
Daniel Svärd
Electronic Devices, Department of Electrical Engineering

ANSWERS
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TSEK37 ANALOG CMOS INTEGRATED CIRCUITS

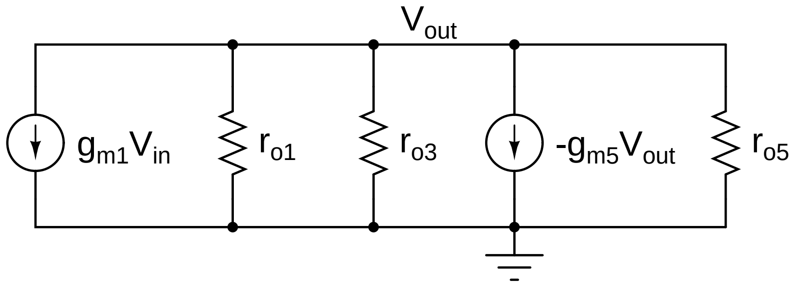
Date:	2013-03-27
Time:	8-12
Location:	U7-U10
Aids:	Calculator, Dictionary
Teachers:	Behzad Mesgarzadeh (5719)
	Daniel Svärd (8946)

8 points are required to pass.

Please start each new problem at the top of a page!
Only use one side of each paper!

1)

(a) Small-signal model:



(b) Find the DC gain:

KCL at V_{out}:

$$g_{m1}V_{in} + (g_{ds1} + g_{ds3} + g_{ds5} - g_{m5})V_{out} = 0 \Rightarrow$$

$$\frac{V_{out}}{V_{in}} = -\frac{g_{m1}}{g_{ds1} + g_{ds3} + g_{ds5} - g_{m5}} = -\frac{g_{m1}\dfrac{r_{o1}r_{o3}r_{o5}}{r_{o1}r_{o3} + r_{o3}r_{o5} + r_{o1}r_{o5}}}{1 - g_{m5}\dfrac{r_{o1}r_{o3}r_{o5}}{r_{o1}r_{o3} + r_{o3}r_{o5} + r_{o1}r_{o5}}} = -\frac{g_{m1}\left(r_{o1}\|r_{o3}\|r_{o5}\right)}{1 - g_{m5}\left(r_{o1}\|r_{o3}\|r_{o5}\right)}$$

(c) What kind of circuit element do M_5 and M_6 realize?

Answer: From (b) we can see that M5(M6) realize a negative resistor. If sized properly this can be used to increase the gain of the circuit.

2)

To have a perfect mirror $V_{DS1} = V_{DS2} = V_Y = 1$ V.
Thus $V_{DS3} = V_{DD} - V_{DS2} = 2 - 1 = 1$ V

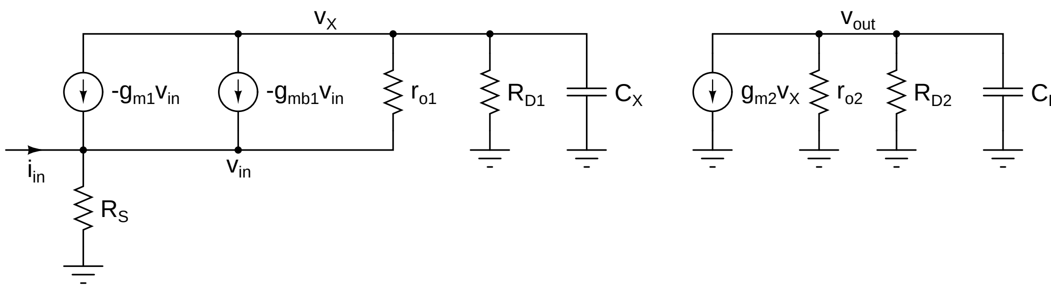
$$I_{D2} = I_{D3} \Rightarrow (1-0.5)^2(1+0.1\times1) = 4\times(V_{GS3}-0.5)^2(1+0.1\times1) \Rightarrow V_{GS3} = 0.75\,\text{V}$$

So: $V_b = 1 + V_{GS3} = 1.75$ V.

Answer: $V_b = 1.75$ V.

3)

(a) Small-signal model:

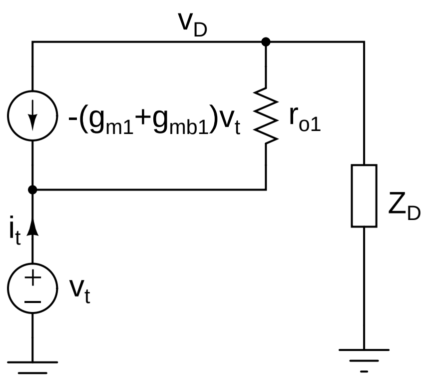


(b) Derive the transfer function. Find expressions for the DC transimpedance and the two poles.

We can divide the transfer function into three separate transfer functions:

$$R(s) = \frac{v_{out}}{i_{in}}(s) = \frac{v_{in}}{i_{in}} \frac{v_X}{v_{in}} \frac{v_{out}}{v_X}(s)$$

The first one is the input impedance of the circuit. It is the parallel combination of the source resistance R_s and the impedance looking into the source of the transistor. To find the latter, we use the following model of the first stage transistor:



We see that $i_t = \frac{v_D}{Z_D}$ where $Z_D = \left(R_{D1} \| \frac{1}{sC_X} \right) = \frac{R_{D1}}{1 + sC_X R_{D1}}$.

And $Z_{in,M1} = \frac{v_t}{i_t} = \frac{v_t}{v_D} Z_D$ (1)

KCL at v_D:

$$\left(\frac{1}{Z_D}+\frac{1}{r_{o1}}\right)v_D=\left(g_{m1}+g_{mb1}+\frac{1}{r_{o1}}\right)v_t \Rightarrow \frac{v_t}{v_D}=\frac{\frac{1}{Z_D}+\frac{1}{r_{o1}}}{g_{m1}+g_{mb1}+\frac{1}{r_{o1}}} \qquad (2)$$

So from (1) and (2):

$$Z_{in,M1}=\frac{1+\frac{Z_D}{r_{o1}}}{g_{m1}+g_{mb1}+\frac{1}{r_{o1}}}=\frac{r_{o1}+Z_D}{1+(g_{m1}+g_{mb1})r_{o1}}=\underbrace{\frac{r_{o1}+R_{D1}}{1+(g_{m1}+g_{mb1})r_{o1}}}_{Z_0}\frac{1+sC_X\left(R_{D1}\|r_{o1}\right)}{1+sC_X R_{D1}} \qquad (3)$$

The input impedance is the parallel combination of R_s and $Z_{in,M1}$

$$Z_{in}=\frac{v_{in}}{i_{in}}=\frac{R_S Z_{in,M1}}{R_S+Z_{in,M1}}=\frac{R_S Z_0}{R_S+Z_0}\frac{1+sC_X\left(R_{D1}\|r_{o1}\right)}{1+sC_X\frac{R_s R_{D1}+Z_0\left(R_{D1}\|r_{o1}\right)}{R_S+Z_0}} \qquad (4)$$

The second transfer function, from v_{in} to v_x is easily found from (2).

$$\frac{v_X}{v_{in}}=\frac{g_{m1}+g_{mb1}+\frac{1}{r_{o1}}}{\frac{1}{Z_D}+\frac{1}{r_{o1}}}=\frac{\left[1+(g_{m1}+g_{mb1})r_{o1}\right]R_{D1}}{r_{o1}+R_{D1}}\frac{1}{1+sC_X\left(R_{D1}\|r_{o1}\right)} \qquad (5)$$

The third transfer function, from v_x to v_{out}, can be found from KCL at v_{out}:

$$\left(\frac{1}{r_{o2}}+\frac{1}{R_{D2}}+sC_L\right)v_{out}=-g_{m2}v_x \Rightarrow \frac{v_{out}}{v_x}=-\frac{g_{m2}\left(R_{D2}\|r_{o2}\right)}{1+sC_L\left(R_{D2}\|r_{o2}\right)} \qquad (6)$$

(4), (5) and (6) then gives the final transfer function as

$$\frac{v_{out}}{i_{in}}=\frac{v_{in}}{i_{in}}\frac{v_x}{v_{in}}\frac{v_{out}}{v_x}=R_0\frac{1}{1+\frac{s}{\omega_{p1}}}\frac{1}{1+\frac{s}{\omega_{p2}}}$$

where,

$$R_0=-\frac{g_{m2}\left(R_{D2}\|r_{o2}\right)\left[1+(g_{m1}+g_{mb1})r_{o1}\right]R_{D1}}{\left[1+(g_{m1}+g_{mb1})r_{o1}\right]R_S+r_{o1}+R_{D1}}\cdot R_S$$

$$\omega_{p1}=\frac{1}{C_L\left(R_{D2}\|r_{o2}\right)}$$

$$\omega_{p2}=\frac{1}{C_X\left(R_{D1}\|\left[\left(1+(g_{m1}+g_{mb1})r_{o1}\right)R_S+r_{o1}\right]\right)}$$

(c) Calculate the value of the DC transimpedance and the poles found in (b).

We need to find the values for g_{m1}, g_{mb1}, r_{o1}, g_{m2} and r_{o2}. We have:

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2(1+\lambda_n V_{DS})$$
$$V_{TH} = V_{t0n} + \gamma\sqrt{V_{SB}+2\Phi_F}$$

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \sqrt{2\mu_n C_{ox}\frac{W}{L} I_D(1+\lambda_n V_{DS})}$$
$$g_{mb} = \frac{\partial I_D}{\partial V_{BS}} = g_m\left(-\frac{\partial V_{TH}}{\partial V_{BS}}\right) = g_m\left(\frac{\partial V_{TH}}{\partial V_{SB}}\right) = g_m\frac{\gamma}{2\sqrt{V_{SB}+2\Phi_F}}$$
$$g_{ds} = \frac{1}{r_o} = \frac{\partial I_D}{\partial V_{DS}} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2\lambda_n = \frac{\lambda_n I_D}{1+\lambda_n V_{DS}}$$

We calculate the parameters we need to find $V_{DS1,}$ V_{SB1} and V_{DS2}.

$$V_{DS1} = V_{dd} - I_1(R_{D1}+R_S) = 1.3\,\text{V}$$

$$V_{SB1} = I_1 R_S = 1\ \text{V}$$

$$V_{DS2} = V_{dd} - I_2 R_{D2} = 2.3\,\text{V}$$

Plugging in the values gives:

g_{m1} = 4.25 mS
g_{mb1} = 0.62 mS
r_{o1} = 22.6 kΩ
g_{m2} = 8.87 mS
r_{o2} = 6.15 kΩ

Using the expressions from (b) yields the following values for the DC transimpedance and the poles:

R_0 = -7385 Ω

ω_{p1} = 1.08 Grad/s = 172.1 MHz

ω_{p2} = 2.02 Grad/s = 320.9 MHz

4)

$$R = r \cdot d = 5\times10^{3} \times 2\times10^{-3} = 10 < 2Z_0 \times \ln(2) = 2\times 50\times 0.69 = 69$$

Then the wire is an LC wire:

$$t_d = \frac{d}{v} = \frac{d}{\frac{C_0}{\sqrt{\varepsilon_r}}} = \frac{2\times10^{-3}}{\frac{3\times10^{8}}{\sqrt{4}}} = 13.4 \text{ ps}$$

5)

The stage gain is $-g_m R_{tot}$, where $R_{tot}= R||r_o$. Circuit has three poles as:

$$\omega_{p1} = \omega_{p2} = \omega_{p3} = \frac{1}{R_{tot}C}$$

Open-loop transfer function: $H(s) = \frac{(-g_m R_{tot})^3}{(1+R_{tot}Cs)^3}$

To calculate the phase margin, we solve $\left|H(j\omega_u)\right| = 1$ to find the unity-gain frequency. Thus:

$$\frac{{g_m}^3 {R_{tot}}^3}{\left|1+j\omega_u R_{tot} C\right|^3} = 1 \rightarrow \omega_u = 9.2\times10^{9} \text{ rad/s}$$

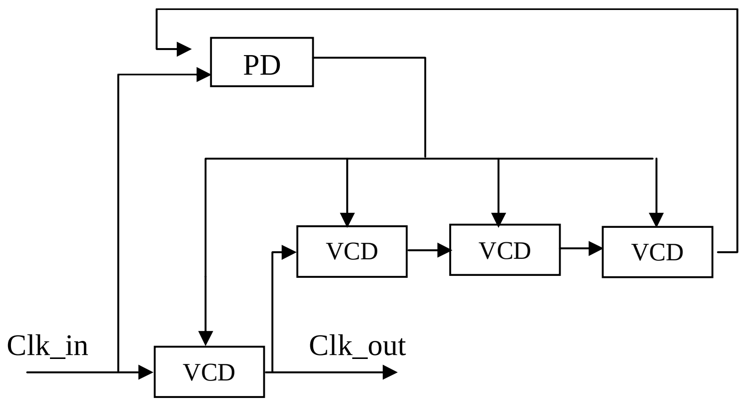
The phase of open-loop transfer function is:

$$-3\tan^{-1}(R_{tot}C\omega_u) = -\tan^{-1}(2.4) = -202^\circ$$

Thus the phase margin is $-202+180=-22^\circ$. Due to this negative phase margin, the feed-back loop is unstable and it oscillates.

6)

A DLL does the job. We need $360^o/90^o = 4$ voltage controlled delay elements (VCDs) in the voltage controlled delay line. The block diagram is shown below.



$$\text{Average} = 1 - \frac{2 \times 45}{360} \times 1 = 0.75 \text{ V}$$