LINKÖPING UNIVERSITY
Behzad Mesgarzadeh
Ali Fazli, Daniel Svärd
Electronic Devices, Department of Electrical Engineering

ANSWERS
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TSEK37 ANALOG CMOS INTEGRATED CIRCUITS

Date:	2012-12-22
Time:	8-12
Location:	U4/U7/U10
Aids:	Calculator, Dictionary
Teachers:	Behzad Mesgarzadeh (5719)
	Ali Fazli (2794)
	Daniel Svärd (8946)

8 points are required to pass.

Please start each new problem at the top of a page!
Only use one side of each paper!

Solution for 1

(a) To determine V_b we first need to know the current through M_5. This is the current through M_1 minus the current through M_3.

$$I_{D3} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_3\left(\frac{V_{DD}}{2} - |V_{t0p}|\right)^2\left(1+|\lambda_p|\frac{V_{DD}}{2}\right) = 1.15\,\text{mA}$$

$$I_{D5} = I_{D1} - I_{D3} = \frac{9.2\,\text{mA}}{2} - 1.15\,\text{mA} = 3.45\,\text{mA}$$

Now we can find the value of V_b as V_{DD} minus the source-gate voltage of M_5.

$$V_{SG5} = \sqrt{\frac{2I_{D5}}{\mu_p C_{ox}\left(\frac{W}{L}\right)_5\left(1+|\lambda_p|\frac{V_{DD}}{2}\right)}} + |V_{t0p}| = 1.5\,\text{V}$$

$$V_b = V_{DD} - V_{SG5} = 3 - 1.5 = 1.5\,\text{V}$$

Answer: V_b is 1.5 V to make the output bias level $V_{DD}/2$.

(b) We find the ratio of g_{m1} to g_{m5}.

$$I_D = \frac{1}{2}\mu_{n/p} C_{ox}(W/L)(V_{GS} - V_{t0})^2(1+\lambda_{n/p}V_{DS})$$

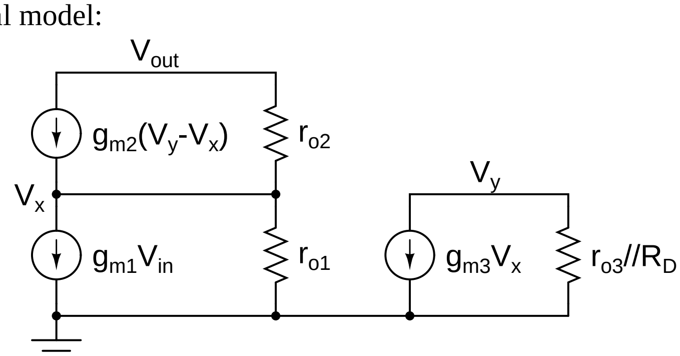
$$g_m = \frac{dI_D}{dV_{GS}} = \sqrt{2\mu_{n/p}C_{ox}(W/L)I_D(1+\lambda_{n/p}V_{DS})}$$

$$\frac{g_{m1}}{g_{m5}} = \sqrt{\frac{\mu_n C_{ox}(W/L)_1 I_{D1}(1+\lambda_n V_{DS1})}{\mu_p C_{ox}(W/L)_5 I_{D5}(1+|\lambda_p|V_{SD5})}} = \sqrt{\frac{200\times 84\times 4.6\times(1+0)}{50\times 120\times 3.45\times(1+0.1\times 1.5)}} \approx 1.8$$

Answer: The ratio g_{m1}/g_{m5} is 1.8.

Solution for 2

(a) Small-signal model:



(b) Find the DC gain.

KCL at V_x:

$$g_{m1}V_{in} + \frac{V_x}{r_{o1}} = g_{m2}(V_y - V_x) + \frac{V_{out} - V_x}{r_{o2}} \Rightarrow g_{m1}V_{in} + \left(g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}\right)V_x = g_{m2}V_y + \frac{V_{out}}{r_{o2}} \tag{1}$$

KCL at V_y:

$$V_y = -g_{m3}(r_{o3} // R_D)V_x \tag{2}$$

KCL at V_{out}:

$$g_{m2}(V_y - V_x) + \frac{V_{out} - V_x}{r_{o2}} = 0 \Rightarrow \frac{V_{out}}{r_{o2}} + g_{m2}V_y = \left(g_{m2} + \frac{1}{r_{o2}}\right)V_x \tag{3}$$

(2) in (1):

$$g_{m1}V_{in} + \left(g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}\right)V_x = -g_{m2}g_{m3}(r_{o3} // R_D)V_x + \frac{V_{out}}{r_{o2}}$$

$$\Rightarrow g_{m1}r_{o1}r_{o2}V_{in} + [(1 + (1 + g_{m3}(r_{o3} // R_D))g_{m2}r_{o2})r_{o1} + r_{o2}]V_x = r_{o1}V_{out} \tag{4}$$

(2) in (3):

$$\frac{V_{out}}{r_{o2}} - g_{m2}g_{m3}(r_{o3} // R_D)V_x = \left(g_{m2} + \frac{1}{r_{o2}}\right)V_x \Rightarrow V_x = \frac{1}{1 + (1 + g_{m3}(r_{o3} // R_D))g_{m2}r_{o2}}V_{out} \tag{5}$$

(5) in (4):

$$g_{m1}r_{o1}r_{o2}V_{in} + \frac{[(1 + (1 + g_{m3}(r_{o3} // R_D))g_{m2}r_{o2})r_{o1} + r_{o2}]}{1 + (1 + g_{m3}(r_{o3} // R_D))g_{m2}r_{o2}}V_{out} = r_{o1}V_{out} \Rightarrow$$

$$g_{m1}r_{o1}r_{o2}[1 + (1 + g_{m3}(r_{o3} // R_D))g_{m2}r_{o2}]V_{in} = -r_{o2}V_{out}$$

Answer: $\frac{V_{out}}{V_{in}} = -g_{m1}r_{o1}[1 + (1 + g_{m3}(r_{o3} // R_D))g_{m2}r_{o2}] \approx -g_{m1}r_{o1} \cdot g_{m2}r_{o2} \cdot g_{m3}(r_{o3} // R_D)$

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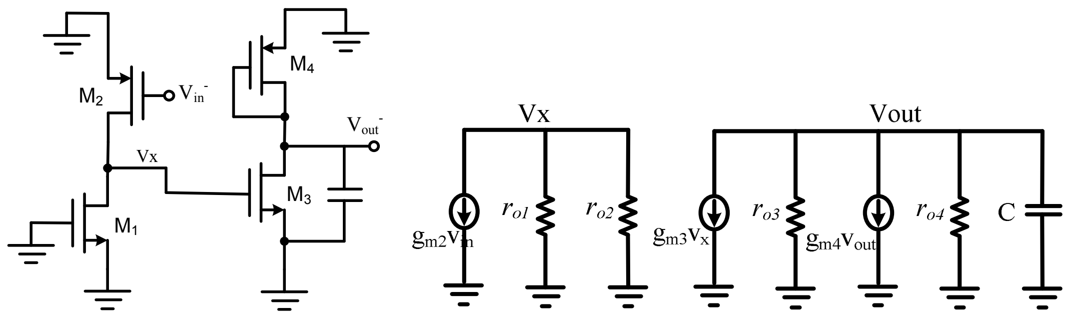
Solution for 3

(a) Apply half-circuit analysis



At V_x: $$g_{m2}Vin+(g_{ds1}+g_{ds2})V_x=0 \Rightarrow \frac{V_x}{Vin}=\frac{-g_{m2}}{g_{ds1}+g_{ds2}} \quad (1)$$

At V_{out}: $$g_{m3}Vx+(g_{m4}+g_{ds3}+g_{ds4}+SC)V_{out}=0 \Rightarrow \frac{V_{out}}{Vx}=\frac{-g_{m3}}{g_{m4}+g_{ds3}+g_{ds4}+sC} \quad (2)$$

Using (1) and (2) we can find out the overall transfer function as the following:

Answer: $$\frac{V_{out}}{Vin}(s)=\frac{-g_{m2}}{g_{ds1}+g_{ds2}}\times\frac{-g_{m3}}{g_{m4}+g_{ds3}+g_{ds4}+sC}=\frac{A_0}{1+s/\omega_0} \quad (3)$$

(b) Applying *s=0* in the equation (3) will provide the DC gain, A_0 as follows

$$A_0=\frac{g_{m2}\cdot g_{m3}}{(g_{ds1}+g_{ds2}).(g_{m4}+g_{ds3}+g_{ds4})} \quad (4)$$

$$\frac{V_{out}}{Vin}(s)=\frac{A_0}{1+s/\omega_0}=\frac{g_{m2}\cdot g_{m3}}{(g_{ds1}+g_{ds2}).(g_{m4}+g_{ds3}+g_{ds4})}\frac{1}{1+s/(\frac{g_{m4}+g_{ds3}+g_{ds4}}{C})} \quad (5)$$

where $$\omega_0=\frac{g_{m4}+g_{ds3}+g_{ds4}}{C}=\frac{g_{m4}+1/ro3+1/ro4}{C}=\frac{1}{(1/gm4//ro3//ro4)C}, \quad (6)$$

therefore, the dominant pole is

Answer: $$f_0=\frac{1}{2\pi(1/gm4//ro3//ro4)C}$$

(c) The unity-gain frequency $f_u \cong A_0.f_0$. Using (4) and (6), f_u can be found out as

$$f_u = \frac{g_{m2}.g_{m3}}{(g_{ds1}+g_{ds2}).(g_{m4}+g_{ds3}+g_{ds4})}.\frac{g_{m4}+g_{ds3}+g_{ds4}}{2\pi.C} = \frac{g_{m2}.g_{m3}}{2\pi.(g_{ds1}+g_{ds2}).C} \qquad (7)$$

Solution for 4

a) The total delay $D(m) = mt_p + m\left[0.38(\frac{d}{m})^2 rc\right] = mt_p + 0.38\frac{d^2}{m}rc$

For optimum delay we should have $\frac{dD(m)}{dm} = t_p - 0.38\frac{d^2}{m^2}rc = 0$

$$m_{opt} = \sqrt{\frac{0.38rcd^2}{t_p}} \qquad (1)$$

$$D(8) = 8t_p + 0.38rc\frac{d^2}{8}$$

$$D(6) = 6t_p + 0.38rc\frac{d^2}{6}$$

$$D(8) - D(6) = 2t_p + 0.38rcd^2(\frac{1}{8} - \frac{1}{6}) = \frac{4t_p}{3}$$

$$t_p = \frac{1}{16}0.38rcd^2 \qquad (2)$$

(1) and (2): $m_{opt} = \sqrt{\frac{0.38rcd^2}{t_p}} = \sqrt{\frac{0.38rcd^2}{\frac{1}{16}0.38rcd^2}} = 4$

Answer: $m_{opt} = 4$

Solution for 5

For a 4-stage ring oscillator (with identical stages), the transfer function is written as:

$$H(j\omega) = -\frac{A^4}{(1 + j\omega/\omega_0)^4}$$

where A is the DC gain of each stage, and ω_0 is the pole frequency. According Barkhausen criteria, the phase contribution of each stage should be 45^o. Then:

$$\tan^{-1}(\omega_{osc}/\omega_0) = 45^o \Rightarrow \omega_{osc} = \omega_0 \qquad (1)$$

Putting the magnitude of the transfer function greater than 1 gives:

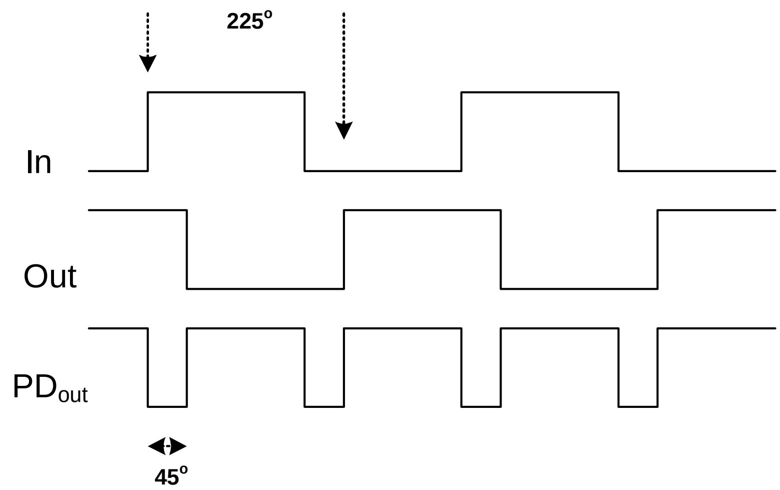
$$|H(j\omega)| = \frac{A^4}{\sqrt{(1 + (\omega/\omega_0)^2)^4}} \geq 1$$

Considering (1), it results is: $|A| \geq \sqrt{2}$

In case of the differential stage the DC gain is $-g_m R$. Then:

$$g_m R \geq \sqrt{2} \Rightarrow R \geq \sqrt{2} \;\; K\Omega$$

Solution for 6



$$\text{Average} = 3 - \frac{2 \times 45}{360} \times 3 = 2.25 \text{ V}$$