TSEK37 – Tutorial 6

Problem 1

Each stage has a transfer function of $-\frac{A}{1+\frac{s}{\omega_o}}$ which gives the total transfer function with 5 stages to be $\frac{V_{out}}{V_{in}} = H^5(s) = -\frac{A^5}{\left(1+\frac{s}{\omega_o}\right)^5}$

To have oscillation according to the Barkhausen criteria the gain has to be larger than 1 when the phase is -180°.

$$\arg\frac{V_{out}}{V_{in}}(j\omega) = -5\arctan\frac{\omega}{\omega_o} = -180°$$

$$\frac{\omega}{\omega_o} = \tan\frac{180°}{5} = \tan 36° = 0.7265$$

Which means that the oscillation frequency $\omega = 0.7264\ \omega_o$ or just below the pole frequency.

If we now look at the magnitude at that frequency

$$|H(j\omega)| = \frac{|-A^5|}{\left(\sqrt{1^2+\left(\frac{\omega}{\omega_o}\right)^2}\right)^5} = 1$$

$$A = \sqrt{1+0.7265^2} = \sqrt{1.5278} = 1.236 \approx 1.24$$

So we have $A_{min} \approx 1.24$.

Problem 2

We know the K_{VCO} and one point, for the other point we can use the one we want to calculate.

$$K_{VCO} = \frac{2\pi 1.25 * 10^9 - 2\pi * 10^9}{x-0.5} = 2\pi 10^8$$

$$x = \frac{2\pi 2.5 * 10^8}{2\pi 10^8} + 0.5 = 2.5 + 0.5 = 3$$

Problem 3

If we start by formulating the KCL of the node by the negative terminal of the OP-amp

$$\frac{V_{in}}{R_2} = -\frac{V_{out}}{R_1 + \frac{1}{sC}}$$

Which gives us

$$\frac{V_{out}}{V_{in}} = -\frac{1+sR_1C}{sR_2C}$$

Problem 4

a) First we derive the closed-loop transfer function

$$\frac{d\theta_{out}}{d\,t} = K_{VCO}V_2 \overset{Laplace}{\Longrightarrow} s\,\theta_{out} = K_{VCO}V_2 \Rightarrow \theta_{out} = \frac{K_{VCO}}{s}V_2$$

$$\theta_{out}(s) = \frac{K_{VCO}}{s}\left(-\frac{1+sR_1C}{sR_2C}\right)K_{PD}\big(\theta_{in}(s) - \theta_{out}(s)\big)$$

$$= -\frac{K_{VCO}K_{PD}}{s^2R_2C}(1+sR_1C)\big(\theta_{in}(s) - \theta_{out}(s)\big)$$

$$[s^2R_2C - K_{VCO}K_{PD}(1+sR_1C)\theta_{out}(s) = -K_{VCO}K_{PD}(1+sR_1C)\theta_{in}(s)$$

$$\frac{\theta_{out}(s)}{\theta_{in}(s)} = -\frac{K_{VCO}K_{PD}(1+sR_1C)}{s^2R_2C - K_{VCO}K_{PD}(1+sR_1C)}$$

b) For stability all poles has to be in the left half-plane.

$$s = \frac{K_{VCO}K_{PD}R_1}{2R_2} \pm \sqrt{\left(\frac{K_{VCO}K_{PD}R_1}{2R_2}\right)^2 + \frac{K_{VCO}K_{PD}}{R_2C}}$$

For both poles to be in the left half-plane, $K_{VCO}K_{PD}$ must be negative.
However this is not enough, one of the following two conditions also has to be fulfilled,

1. The expression under the root sign has to be negative thus introducing complex poles.
2. The expression has to have a value less than $\left(\frac{K_{VCO}K_{PD}R_1}{2R_2}\right)^2$ in order to keep both poles in the left half-plane. (if $K_{VCO}K_{PD} < 0$ then this is true)