Exercises for Tutorial 4: Feedback, Stability, Frequency Compensation

1) Problem 10.1 in the course book.

We get a transfer function $A_{open}(s) = \frac{A_0}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)}$, where A_0 is unknown.

The closed loop TF is $A_{closed}(s) = \frac{A_{open}(s)}{1+\beta A_{open}(s)}$

The phase margin is defined as $PM = 180° + \arg[\beta A_{open}(s = j\omega_u)]$

Since the phase margin should be 60° we can write

$$180° - 60° = 120° = -\arg[\beta A_{open}(s = j\omega_u)] = \arctan\left[\frac{\omega_u\left(\frac{1}{\omega_{p1}}+\frac{1}{\omega_{p2}}\right)}{1-\frac{\omega_u^2}{\omega_{p1}\omega_{p2}}}\right] \rightarrow$$

$$\omega_u\left(\frac{1}{\omega_{p1}}+\frac{1}{\omega_{p2}}\right) = \tan(120°) - \tan(120°)\frac{\omega_u^2}{\omega_{p1}\omega_{p2}} \rightarrow$$

$$\omega_u = -\frac{\omega_{p1}+\omega_{p2}}{2\tan(120°)} \pm \sqrt{\left(\frac{\omega_{p1}+\omega_{p2}}{2\tan(120°)}\right)^2 + \omega_{p1}\omega_{p2}} \approx 310.5\ MHz$$

$$|\beta A_{open}(s = j\omega_u)| = \frac{A_0}{\sqrt{\left(1-\frac{\omega_u^2}{\omega_{p1}\omega_{p2}}\right)^2 + \omega_u^2\left(\frac{1}{\omega_{p1}}+\frac{1}{\omega_{p2}}\right)^2}} = 1 \rightarrow$$

$$A_0 = \sqrt{\left(1-\frac{\omega_u^2}{\omega_{p1}\omega_{p2}}\right)^2 + \omega_u^2\left(\frac{1}{\omega_{p1}}+\frac{1}{\omega_{p2}}\right)^2} \approx 36.6 \approx 31.3\ dB$$

2) Problem 10.9 in the course book. In part (b) assume that the gain crossover point is the same as that of part (a). Also assume $\mu_n C_{ox} = 134\ \mu A/V^2$, $\lambda_n = 0.1\ V^{-1}$ and $\lambda_p = 0.2\ V^{-1}$. All transistors are in saturation region.

3) Figure 8 shows an amplifier schematic. For simplicity we can ignore all parasitics of M_1 and M_2. Also, we assume $g_{m1} \gg 1/r_{o1}$ and $\gamma = 0$.

a) Determine the transfer function of the amplifier.

b) If $g_{m1} = g_{m2} = 1\frac{mA}{V}$, $R = r_{o2} = 20\ k\Omega$ and $C_1 = C_2 = 1\ pF$, calculate the phase margin of the circuit.

$$PM \quad 180° + \arg(H(j\omega)), |H(j\omega)| = 1$$

$$|H(j\omega_u)| = \frac{|A_0|}{\left|\left(1+j\frac{\omega_u}{\omega_{p1}}\right)\left(1+j\frac{\omega_u}{\omega_{p2}}\right)\right|} = 1 \rightarrow$$

$$\omega_u^2 = x$$

$$(\omega_{p1}\omega_{p2} - x)^2 + x(\omega_{p1}+\omega_{p2})^2 = A_0^2(\omega_{p1}\omega_{p2})^2 \rightarrow$$

$$x = -\frac{\omega_{p1}^2+\omega_{p2}^2}{2} \pm \sqrt{(A_0^2-1)(\omega_{p1}\omega_{p2})^2 + \left(\frac{\omega_{p1}^2+\omega_{p2}^2}{2}\right)^2} = \begin{cases} 6.11e17 \leftarrow OK \\ -1.62e18 \end{cases}$$

$$\omega_u = \pm\sqrt{x} = 781e6 \left[\frac{rad}{s}\right]$$

$$\arg(H(j\omega_u)) = \arctan\left[-\frac{\omega_u(\omega_{p1}+\omega_{p2})}{\omega_{p1}\omega_{p2}-\omega_u^2}\right] = -120.7°$$

c) Use the assumptions in part (b) to calculate the AC gain, if the input frequency is $f = \frac{1}{2\pi} \times 10^8\ Hz$

$$\omega_{in} = \omega_{p1} = 10^8 \frac{rad}{s}$$

$$\frac{\omega_{p1}}{\omega_{p2}} = \frac{10^8}{10^9} = 0.1$$

$$|H(j\omega_{p1})| = \frac{|A_0|}{\sqrt{\left(1-\frac{\omega_{p1}}{\omega_{p2}}\right)^2 + \left(1+\frac{\omega_{p1}}{\omega_{p2}}\right)^2}} = \frac{10}{\sqrt{0.9^2+1.1^2}} = \frac{10}{\sqrt{2.02}} \approx 7$$

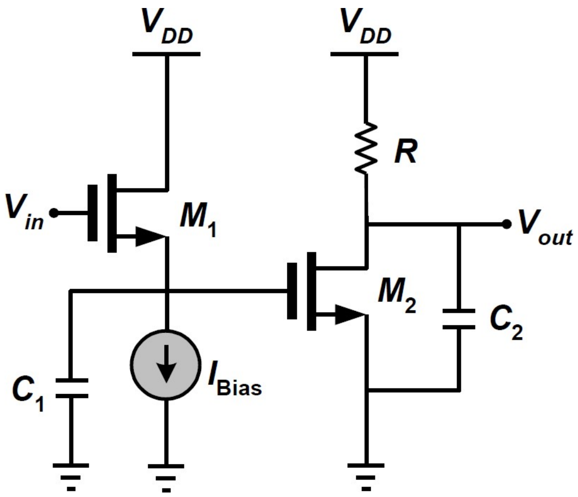

Figure 8 An amplifier schematic.

4) Figure 9 shows an amplifier schematic. For simplicity we can ignore all parasitics of M_1 and M_2. Also we assume $\lambda = 0$.

a) Determine the transfer function of the amplifier.

b) If the amplifier behaves like a single-pole system, show $g_{m2}R = 1$.

$$g_{m2}R = 1$$

$$\frac{V_{out}}{V_{in}} = -\frac{g_{m1}R(1+sC_1R)}{(1+sC_1R)(1+sC_2R)} = -\frac{g_{m1}R}{1+sC_2R}, \text{ where } \omega_p = \frac{1}{C_2R}$$

c) If $g_{m1} = g_{m2} = 0.32\frac{mA}{V}$, $R = 5\ k\Omega$, $C_1 = 0.2\ pF$ and $C_2 = 1\ pF$, calculate the phase shift through the amplifier circuit for an input signal with $f = 143.3\ MHz$.

$$\omega_{in} = 2\pi f_{in} = 2\pi 143.3e6 \left[\frac{rad}{s}\right]$$

$$H(j\omega) = -\frac{g_{m1}}{(g_{m2}-\omega^2RC_1C_2)^2 + [\omega(C_1+C_2)]^2}([g_{m2}+\omega^2C_1^2R] + j\omega[RC_1(g_{m2}-\omega^2C_1C_2R) - C1 + C_2])$$

$$\arg(H(j\omega)) = \arctan\left[\frac{\omega[RC_1(g_{m2} - \omega^2 RC_1C_2) - C_1 - C_2]}{g_{m2} + \omega^2 C_1^2 R}\right] \approx -72.4°$$

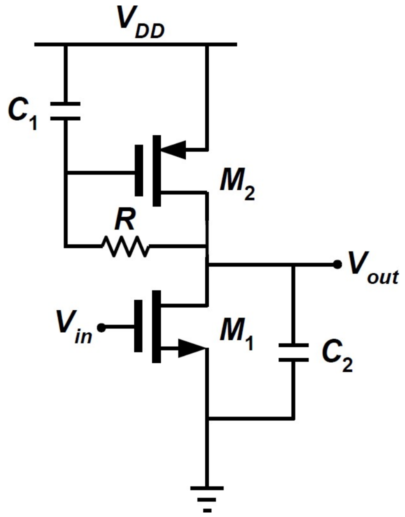

Figure 9 An amplifier schematic.

5) An amplifier circuit has two poles at $100\frac{Mrad}{s}$ and $500\frac{Mrad}{s}$, with no zeros. Calculate the DC gain of the amplifier to get a phase margin of 90°.

$$\arg(H(j\omega)) = \arctan\left[-\frac{\omega_u(\omega_{p1} + \omega_{p2})}{\omega_{p1}\omega_{p2} - \omega_u^2}\right] = -90°$$

$$-\frac{\omega_u(\omega_{p1} + \omega_{p2})}{\omega_{p1}\omega_{p2} - \omega_u^2} = \tan(-90°) \approx -\infty \rightarrow$$

$$\omega_{p1}\omega_{p2} - \omega_u^2 \approx 0 \rightarrow \omega_u = \sqrt{\omega_{p1}\omega_{p2}} \approx 224\frac{Mrad}{s}$$

$$|H(j\omega_u)| = \left|\frac{A_0\omega_{p1}\omega_{p2}}{(\omega_{p1}\omega_{p2} - \omega_u^2) + j\omega_u(\omega_{p1} + \omega_{p2})}\right| = 1 \rightarrow$$

$$A_0 = \frac{\sqrt{(\omega_{p1}\omega_{p2} - \omega_u^2)^2 + \omega_u^2(\omega_{p1} + \omega_{p2})^2}}{\omega_{p1}\omega_{p2}} = \frac{\sqrt{0^2 + \omega_u^2(\omega_{p1} + \omega_{p2}^2)}}{\omega_{p1}\omega_{p2}} =$$

$$\frac{\omega_u(\omega_{p1} + \omega_{p2})}{\omega_{p1}\omega_{p2}} = \frac{\omega_u(\omega_{p1} + \omega_{p2})}{\omega_u^2} = \frac{\omega_{p1} + \omega_{p2}}{\omega_u} = \frac{\omega_{p1} + \omega_{p2}}{\sqrt{\omega_{p1}\omega_{p2}}} \approx 2.68 \approx 8.6\ dB$$

6) Figure 10 shows a source-follower circuit. For simplicity we can ignore all parasitics. Also we assume $\lambda = 0$.

a) Determine the transfer function of the circuit.

b) If $g_{m1} = 1\frac{mA}{V}$, $R = 10\ k\Omega$, $C_1 = 1\ pF$ and $C_2 = 0.1\ pF$, calculate the AC gain and the phase shift through the source-follower circuit for an input frequency of $5\frac{Grad}{s}$.

$$|H(j\omega_{in})| = \frac{\sqrt{1+\left(\omega_{in}\frac{C_1(1+g_mR)}{g_m}\right)^2}}{\sqrt{\left(1-\omega_{in}^2\frac{RC_1C_2}{g_m}\right)^2+\left(\omega_{in}\frac{C_1(1+g_mR)+C_2}{g_m}\right)^2}} \approx \frac{55.0}{60.47} \approx 0.91$$

$$\arg(H(j\omega_{in}))$$
$$= \arctan\left[\frac{\omega_{in}\left((1+g_mR)C_1(g_m-\omega_{in}^2RC_1C_2)-g_m((1+g_mR)C_1+C_2)\right)}{g_{m2}-\omega_{in}^2g_mR\,C_1C_2+\omega_{in}^2(1+g_mR)C_1((1+g_mR)C_1+C_2)}\right]$$
$$\approx \arctan\left[-\frac{0.013755}{0.0030285}\right] \approx 24.4°$$

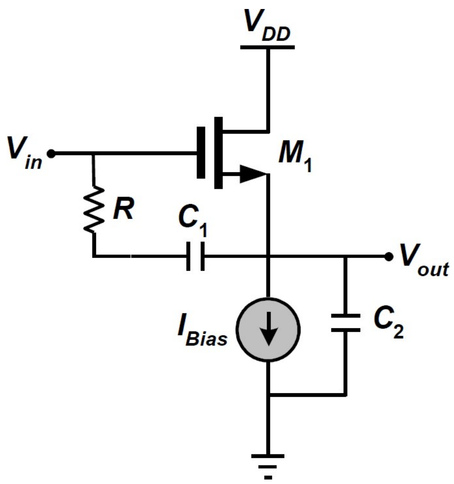

Figure 10 Source follower