



## Exercises for Tutorial 2: Differential Amplifiers

1. Problem 4.18 in the course book (Only for Fig. 4.38. Assume  $\gamma = 0$ . For Fig. 4.38(a) assume  $r_o \gg R_1$  and  $r_o \gg 1/g_m$ . Also in Fig. 4.38(e), assume  $\lambda = 0$ ).

a)

$$V_{out} \left( g_{m3} + \frac{1}{r_{o1}} + \frac{1}{r_{o3}} + \frac{2}{R_1} \right) = -g_{m1} V_{in} \leftrightarrow$$

$$V_{out} \left( g_{m3} + \frac{2}{R_1} \right) = -g_{m1} V_{in} \leftrightarrow V_{out} (2 + g_{m3} R_1) = -g_{m1} R_1 V_{in}$$

$$A_v = \frac{V_{out}}{V_{in}} = -\frac{g_{m1} R_1}{2 + g_{m3} R_1} \text{ or } A_v = \frac{g_{m1} R_1}{2 + g_{m3} R_1}$$

b)

$$g_{m3} V_y + \frac{V_y}{R_1} + \frac{V_y - V_{out}}{r_{o3}} = 0 \leftrightarrow V_y \left( g_{m3} + \frac{1}{r_{o3}} + \frac{1}{R_1} \right) = V_{out} \frac{1}{r_{o3}} \quad (1)$$

$$g_{m1} V_{in} - g_{m3} V_y + \frac{V_{out}}{r_{o1}} + \frac{V_{out} - V_y}{r_{o3}} = 0 \leftrightarrow$$

$$V_{out} \left( \frac{1}{r_{o1}} + \frac{1}{r_{o3}} \right) - V_y \left( g_{m3} + \frac{1}{r_{o3}} \right) = -g_{m1} V_{in} \quad (2)$$

(1) in (2):

$$V_{out} \frac{\left( \frac{1}{r_{o1}} + \frac{1}{r_{o3}} \right) \left( g_{m3} + \frac{1}{r_{o3}} + \frac{1}{R_1} \right) - \frac{1}{r_{o3}} \left( g_{m3} + \frac{1}{r_{o3}} \right)}{g_{m3} + \frac{1}{r_{o3}} + \frac{1}{R_1}} = -g_{m1} V_{in} \leftrightarrow$$

$$A_v = \frac{V_{out}}{V_{in}} = -g_{m1} \frac{r_{o1}(r_{o3} + R_1 + g_{m3} r_{o3} R_1)}{r_{o1} + (r_{o3} + R_1 + g_{m3} r_{o3} R_1)} = -g_{m1} (r_{o1} \parallel (r_{o3} + R_1 + g_{m3} r_{o3} R_1))$$

c)

$$\frac{V_{out}}{r_{o1} \parallel r_{o3} \parallel \frac{R_1}{2}} = -g_{m1} V_{in}$$

$$A_v = \frac{V_{out}}{V_{in}} = -g_{m1} \left( r_{o1} \parallel r_{o3} \parallel \frac{R_1}{2} \right)$$

d)

$$\frac{V_{out}}{r_{o1} \parallel r_{o3} \parallel R_1} = -g_{m1} V_{in} \leftrightarrow A_v = \frac{V_{out}}{V_{in}} = -g_{m1} (r_{o1} \parallel r_{o3} \parallel R_1)$$

e)

$$g_{m1} V_{in} + \frac{V_x - V_{out}}{R_1} = 0 \leftrightarrow V_x = -g_{m1} R_1 V_{in} + V_{out} \quad (1)$$

$$g_{m3} V_x + \frac{V_{out} - V_x}{R_1} = 0 \leftrightarrow V_{out} = (1 - g_{m3} R_1) V_x \quad (2)$$

(1) in (2)  $V_{out} (1 - (1 + g_{m3} R_1)) = -g_{m3} R_1 V_{out} = -g_{m1} R_1 (1 - g_{m3} R_1) V_{in}$

$$A_v = \frac{V_{out}}{V_{in}} = -g_{m1} R_1 \frac{1 - g_{m3} R_1}{g_{m3} R_1} = -g_{m1} \frac{1 - g_{m3} R_1}{g_{m3}} = -\frac{g_{m1}}{g_{m3}} (1 - g_{m3} R_1)$$

2. Problem 4.22 in the course book.

KCL:

$$g_m(V_{in1} - V_x) + \frac{V_{out1} - V_x}{R_p} + \frac{V_{out1}}{R_D} = 0 \Leftrightarrow V_x \left( g_m + \frac{1}{R_p} \right) = g_m V_{in1} + \left( \frac{1}{R_p} + \frac{1}{R_D} \right) V_{out1} \quad (1)$$

$$g_m(V_{in2} - V_x) + \frac{V_{out2}}{R_D} = 0 \Leftrightarrow V_{out2} = -g_m R_D V_{in2} + g_m R_D V_x \quad (2)$$

(1) in (2):

$$V_{out2} = -g_m R_D V_{in2} + \frac{g_m R_D}{g_m + \frac{1}{R_p}} \left( g_m V_{in1} + \left( \frac{1}{R_D} + \frac{1}{R_p} \right) V_{out1} \right) \Leftrightarrow$$

$$(g_m R_p + 1) V_{out2} - (g_m R_p + g_m R_D) V_{out1} = g_m^2 R_p R_D (V_{in1} - V_{in2}) - g_m R_D V_{in2}$$

Let  $V_{out1} + V_{out2} = 0$  and  $V_{out} = V_{out1} - V_{out2} \Leftrightarrow V_{out1} = \frac{V_{out}}{2}, V_{out2} = -\frac{V_{out}}{2}$  and

$$V_{in2} = \frac{V_{in1} + V_{in2}}{2} - \frac{V_{in1} - V_{in2}}{2}$$

$$\left( g_m R_p + \frac{1}{2} + \frac{g_m R_D}{2} \right) V_{out} = -g_m R_D \left( g_m R_p + \frac{1}{2} \right) (V_{in1} - V_{in2}) + \frac{g_m R_D}{2} (V_{in1} + V_{in2}) \Leftrightarrow$$

$$\left( \frac{1}{R_D} + \frac{1}{2R_p} + \frac{1}{2g_m R_p R_D} \right) V_{out} = - \left( g_m + \frac{1}{2R_p} \right) (V_{in1} - V_{in2}) + \frac{V_{in1} + V_{in2}}{2R_p}$$

3. Problem 5.3(a) in the course book for  $\gamma = 0$ . Assume all the transistors are in the saturation region and  $\mu_n C_{ox} = 4\mu_p C_{ox} = 200 \mu A/V^2$ ,  $V_{tn} = |V_{tp}| = 0.5 V$  and  $V_{DD} = 2.5 V$ .

Calculate  $V_p$ :

$$V_{GS1} = \sqrt{\frac{2I_{D1}}{\mu_n C_{ox} \frac{W}{L}}} + V_{tn} = \sqrt{\frac{5I_{ref}}{\mu_n C_{ox} \left( \frac{W}{L} \right)_N}} + V_{tn} \approx 0.85 V$$

$$V_p = V_{in,CM} - V_{GS1} \approx 1.3 V - 0.85 V = 0.45 V$$

Calculate diode-connected PMOS's drain voltages

Mirror:

$$V_D = V_{DD} - |V_{GS1}| = V_{DD} - \left| \sqrt{\frac{4I_{ref}}{\mu_p C_{ox} \left( \frac{W}{L} \right)_p}} + V_{tp} \right| \approx 1.37 V$$

Load:

$$V_D = V_{DD} - |V_{GS1}| = V_{DD} - \left| \sqrt{\frac{2I_D}{\mu_p C_{ox} \left( \frac{W}{L} \right)_p}} + V_{tp} \right| = V_{DD} - \left| \sqrt{\frac{2 \left( \frac{5}{2} I_{ref} - 2I_{ref} \right)}{\mu_p C_{ox} \left( \frac{W}{L} \right)_p}} + V_{tp} \right| \approx 1.68 V$$

4. Calculate the small-signal voltage gain of the cascade differential pair shown in Figure 3. Assume  $\gamma = 0$ .

We use the equivalent half-circuit since the circuit is symmetric.

The Norton equivalent amplifier is a current source in parallel with an output impedance, its gain is  $\frac{V_{out}}{V_{in}} = G_m R_{out}$  where  $G_m = \frac{I_{out}}{V_{in}}$

We find  $G_m$  by connecting  $V_{out}$  to ground and finding  $I_{out}$ .

$$I_{out} = -g_{m3}V_x - \frac{V_x}{r_{o3}} = -\left(g_{m3} + \frac{1}{r_{o3}}\right)V_x \leftrightarrow V_x = -\frac{I_{out}}{g_{m3} + \frac{1}{r_{o3}}} = -\left(\frac{1}{g_{m3}} \parallel r_{o3}\right)I_{out} \quad (1)$$

$$I_{out} = g_{m1}V_{in} + \frac{V_x}{r_{o1}} \leftrightarrow V_x = (I_{out} - g_{m1}V_{in})r_{o1} \quad (2)$$

(1) in (2):

$$(I_{out} - g_{m1}V_{in})r_{o1} = -\left(\frac{1}{g_{m3}} \parallel r_{o3}\right)I_{out} \leftrightarrow I_{out} \left(r_{o1} + \left(\frac{1}{g_{m3}} \parallel r_{o3}\right)\right) = g_{m1}r_{o1}V_{in} \leftrightarrow$$

$$G_m = \frac{I_{out}}{V_{in}} = \frac{g_{m1}r_{o1}}{r_{o1} + \frac{1}{g_{m3}} \parallel r_{o3}} \approx \frac{g_{m1}r_{o1}}{r_{o1}} = g_{m1}$$

Find  $R_{out}$  by shorting the input and measuring  $\frac{V_{test}}{I_{test}}$ :

$$\begin{aligned} I_{test} &= -g_{m3}V_x + \frac{V_{test} - V_x}{r_{o3}} - g_{m5}V_y + \frac{V_{test} - V_y}{r_{o5}} \\ &= -\left(g_{m3} + \frac{1}{r_{o3}}\right)V_x - \left(g_{m5} + \frac{1}{r_{o5}}\right)V_y + \left(\frac{1}{r_{o3}} + \frac{1}{r_{o5}}\right)V_{test} \end{aligned}$$

$$\frac{V_x}{r_{o1}} + g_{m3}V_x + \frac{V_x - V_{test}}{r_{o3}} = 0 \leftrightarrow V_x = \frac{\frac{1}{r_{o3}}}{g_{m3} + \frac{1}{r_{o1}} + \frac{1}{r_{o3}}} V_{test}$$

$$\frac{V_y}{r_{o7}} + g_{m5}V_y + \frac{V_y - V_{test}}{r_{o5}} = 0 \leftrightarrow V_y = \frac{\frac{1}{r_{o5}}}{g_{m5} + \frac{1}{r_{o5}} + \frac{1}{r_{o7}}} V_{test}$$

$$\frac{I_x}{V_{test}} = -\frac{g_{m3}}{r_{o3}} \left(g_{m3} + \frac{1}{r_{o1}} + \frac{1}{r_{o3}}\right)^{-1} + \frac{1}{r_{o3}} \left(1 - \frac{1}{r_{o3}} \left(g_{m3} + \frac{1}{r_{o1}} + \frac{1}{r_{o3}}\right)^{-1}\right)$$

$$\frac{I_x}{V_{out}} = \frac{1}{r_{o1}r_{o3} \left(g_{m3} + \frac{1}{r_{o1}} + \frac{1}{r_{o3}}\right)} = \frac{1}{g_{m3}r_{o1}r_{o3} + r_{o1} + r_{o3}}$$

Symmetry gives that:

$$\frac{I_y}{V_{out}} = \frac{1}{g_{m5}r_{o5}r_{o7} + r_{o5} + r_{o7}}$$

$$\frac{I_{out}}{V_{out}} = \frac{I_x}{V_{out}} + \frac{I_y}{V_{out}} = \frac{1}{g_{m3}r_{o1}r_{o3} + r_{o1} + r_{o3}} + \frac{1}{g_{m5}r_{o5}r_{o7} + r_{o5} + r_{o7}}$$

$$R_{out} = \frac{1}{\frac{I_{out}}{V_{out}}} = (g_{m3}r_{o1}r_{o3} + r_{o1} + r_{o3}) \parallel (g_{m5}r_{o5}r_{o7} + r_{o5} + r_{o7}) \approx (g_{m3}r_{o1}r_{o3}) \parallel (g_{m5}r_{o5}r_{o7})$$

Combining the calculated  $G_m$  with  $R_{out}$  gives:

$$A_v = -G_m R_{out} = -g_{m1}(g_{m3}r_{o1}r_{o3} \parallel g_{m5}r_{o5}r_{o7})$$

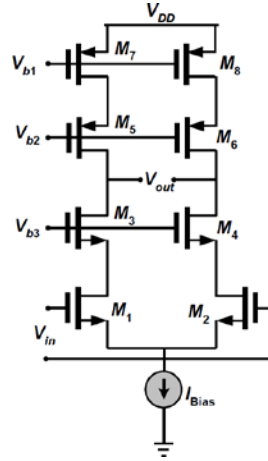


Figure 1 A differential amplifier.

5. Consider the differential amplifier shown in Figure 4. Due to a manufacturing defect, a large parasitic resistance has appeared between the drains of  $M_1$  and  $M_4$ . Assume  $\lambda = \gamma = 0$ . Calculate the small-signal gain, common-mode gain, and CMRR. Assume that  $(W/L)_1 = (W/L)_2$  and  $(W/L)_3 = (W/L)_4$ .

First do an KCL for the ground nodes, remember that  $g_{m1} = g_{m2}$  since  $V_{GS1} = V_{GS2}$  and  $\frac{W}{L}_1 = \frac{W}{L}_2$ :

$$\frac{V_{out1}}{R_D} + \frac{V_{out2}}{R_D} + g_{m1}\Delta V_{in} - g_{m1}\Delta V_{in} = 0 \leftrightarrow V_{out1} = -V_{out2}$$

Then do a KCL at  $V_{out1}$ :

$$\frac{V_{out1}}{R_D} - g_{m3}V_x = 0 \leftrightarrow V_x = \frac{V_{out1}}{g_{m3}R_D}$$

And a KCL at  $V_x$ :

$$g_{m3}V_x + g_{m1}\Delta V_{in} - \frac{V_{out2} - V_x}{R_P} = 0 \leftrightarrow V_{out2} \left( \frac{1}{R_P} \right) = V_x \left( g_{m3} + \frac{1}{R_P} \right) + g_{m1}\Delta V_{in} \leftrightarrow$$

$$V_{out2} = \frac{V_{out1}R_P}{g_{m3}R_D} \left( g_{m3} + \frac{1}{R_P} \right) + g_{m1}R_P\Delta V_{in} \leftrightarrow V_{out1} \left( 1 + \frac{R_P}{R_D} + \frac{1}{g_{m3}R_D} \right) = -g_{m1}R_P\Delta V_{in} \leftrightarrow$$

$$\frac{V_{out1}}{\Delta V_{in}} = -\frac{g_{m1}R_P}{1 + \frac{R_P}{R_D} + \frac{1}{g_{m3}R_D}}$$

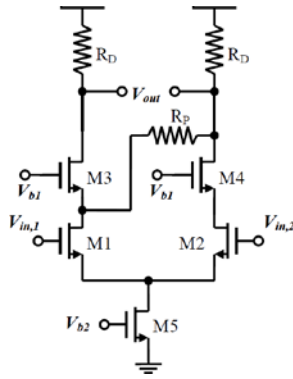


Figure 2 A differential amplifier.

6. A simple current mirror is shown in Figure 5. Calculate the value of  $V_{bias}$  in order to have  $V_N = V_{DD}/2$ . Using this value calculate the error percentage of mirroring. Error percentage can be defined as  $E(\%) = \frac{|I_{ref} - I_{out}|}{I_{ref}} \times 100$ .

$$\mu_n C_{ox} = 200 \frac{\mu A}{V^2}$$

$$\mu_p C_{ox} = 50 \frac{\mu A}{V^2}$$

$$V_{DD} = 3 V$$

$$\left(\frac{W}{L}\right)_3 = 4 \left(\frac{W}{L}\right)_1 = 4 \left(\frac{W}{L}\right)_2 = 20$$

$$R = 1 k\Omega$$

$$V_{tn} = |V_{tp}| = 0.5 V$$

$$\lambda_n = |\lambda_p| = 0.1 V^{-1}$$

Compute  $V_{bias}$  so that  $V_N = \frac{V_{DD}}{2}$ . Assume saturation.

$$M3: I_{ref} = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_3 (V_{DD} - V_{bias} - |V_{tp}|)^2 (1 + |\lambda_p|(V_{DD} - V_N)) \quad (1)$$

$$M1: I_{ref} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_N - V_{tn})^2 (1 + \lambda_n V_N) \quad (2)$$

Divide (1) by (2):

$$1 = \frac{\mu_p C_{ox} \left(\frac{W}{L}\right)_3 (V_{DD} - V_{bias} - |V_{tp}|)^2 (1 + |\lambda_p|(V_{DD} - V_N))}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_N - V_{tn})^2 (1 + \lambda_n V_N)} \leftrightarrow$$

$$1 = \left(\frac{V_{DD} - V_{bias} - |V_{tp}|}{V_N - V_{tn}}\right)^2 \leftrightarrow V_{bias} = V_{DD} - V_N + V_{tn} - |V_{tp}| = V_{DD} - V_N = \frac{V_{DD}}{2}$$

Compute the error percentage  $E(\%) = \frac{|I_{ref} - I_{out}|}{I_{ref}}$

$$I_{out} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_2 (V_N - V_{tn})^2 (1 + \lambda_n (V_{DD} - I_{out}R)) \leftrightarrow$$

$$I_{out} \left[1 + \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_2 (V_N - V_{tn})^2 \lambda_n R\right] = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_2 (V_N - V_{tn})^2 (1 + \lambda_n V_{DD}) \leftrightarrow$$

$$I_{out} = \frac{\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_2 (V_N - V_{tn})^2 (1 + \lambda_n V_{DD})}{1 + \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_2 (V_N - V_{tn})^2 \lambda_n R}$$

$$\frac{I_{out}}{I_{ref}} = \frac{1 + \lambda_n V_{DD}}{1 + \lambda_n V_N} \frac{1}{1 + \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_2 (V_N - V_{tn})^2 \lambda_n R} \approx 1.0766$$

$$E(\%) \approx 7.66 \%$$

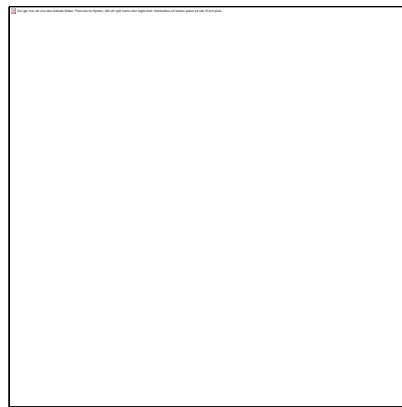


Figure 3 Simple current mirror.

