Exercises for Tutorial 2: Differential Amplifiers

1. Problem 4.18 in the course book (Only for Fig. 4.38. Assume $\gamma = 0$. For Fig. 4.38(a) assume $r_o \gg R_1$ and $r_o \gg 1/g_m$. Also in Fig. 4.38(e), assume $\lambda = 0$).

a)

$$V_{out}\left(g_{m3} + \frac{1}{ro_1} + \frac{1}{r_{o3}} + \frac{2}{R_1}\right) = -g_{m1}V_{in} \leftrightarrow$$

$$V_{out}\left(g_{m3} + \frac{2}{R_1}\right) = -g_{m1}V_{in} \leftrightarrow V_{out}(2 + g_{m3}R_1) = -g_{m1}R_1V_{in}$$

$$A_v = \frac{V_{out}}{V_{in}} = -\frac{g_{m1}R_1}{2 + g_{m3}R_1} \; or \; A_v = \frac{g_{m1}R_1}{2 + g_{m3}R_1}$$

b)

$$g_{m3}V_y + \frac{V_y}{R_1} + \frac{V_y - V_{out}}{r_{o3}} = 0 \leftrightarrow V_y\left(g_{m3} + \frac{1}{r_{o3}} + \frac{1}{R_1}\right) = V_{out}\frac{1}{r_{o3}} \; (1)$$

$$g_{m1}V_{in} - g_{m3}V_y + \frac{V_{out}}{r_{o1}} + \frac{V_{out} - V_y}{r_{o3}} = 0 \leftrightarrow$$

$$V_{out}\left(\frac{1}{r_{o1}} + \frac{1}{r_{o3}}\right) - V_y\left(g_{m3} + \frac{1}{r_{o3}}\right) = -g_{m1}V_{in} \; (2)$$

(1) in (2):

$$V_{out}\frac{\left(\frac{1}{r_{o1}} + \frac{1}{r_{o3}}\right)\left(g_{m3} + \frac{1}{r_{o3}} + \frac{1}{R_1}\right) - \frac{1}{r_{o3}}\left(g_{m3} + \frac{1}{r_{o3}}\right)}{g_{m3} + \frac{1}{r_{o3}} + \frac{1}{R_1}} = -g_{m1}V_{in} \leftrightarrow$$

$$A_v = \frac{V_{out}}{V_{in}} = -g_{m1}\frac{r_{o1}(r_{o3} + R_1 + g_{m3}r_{o3}R_1)}{r_{o1} + (r_{o3} + R_1 + g_{m3}r_{o3}R_1)} = -g_{m1}\big(r_{o1} \parallel (r_{o3} + R_1 + g_{m3}r_{o3}R_1)\big)$$

c)

$$\frac{V_{out}}{r_{o1} \parallel r_{o3} \parallel \frac{R_1}{2}} = -g_{m1}V_{in}$$

$$\mathrm{A_v} = \frac{\mathrm{V_{out}}}{\mathrm{V_{in}}} = -\mathrm{g_{m1}}\left(\mathrm{r_{o1}} \parallel \mathrm{r_{o3}} \parallel \frac{\mathrm{R_1}}{2}\right)$$

d)

$$\frac{V_{out}}{r_{o1} \parallel r_{o3} \parallel R_1} = -g_{m1}V_{in} \leftrightarrow A_v = \frac{V_{out}}{V_{in}} = -g_{m1}(r_{o1} \parallel r_{o3} \parallel R_1)$$

e)

$$g_{m1}V_{in} + \frac{V_x - V_{out}}{R_1} = 0 \leftrightarrow V_x = -g_{m1}R_1V_{in} + V_{out} \; (1)$$

$$\mathrm{g_{m3}V_x} + \frac{\mathrm{V_{out} - V_x}}{\mathrm{R_1}} = 0 \leftrightarrow \mathrm{V_{out}} = (1 - \mathrm{g_{m3}R_1})\mathrm{V_x} \; (2)$$

(1) in (2) $V_{out}\big(1 - (1 + g_{m3}R_1)\big) = -g_{m3}R_1V_{out} = -\mathrm{g_{m1}R_1(1 - g_{m3}R_1)V_{in}}$

$$\mathrm{A_v} = \frac{\mathrm{V_{out}}}{\mathrm{V_{in}}} = -\mathrm{g_{m1}R_1}\frac{1 - \mathrm{g_{m3}R_1}}{\mathrm{g_{m3}R_1}} = -\mathrm{g_{m1}}\frac{1 - \mathrm{g_{m3}R_1}}{\mathrm{g_{m3}}} = -\frac{\mathrm{g_{m1}}}{\mathrm{g_{m3}}}(1 - \mathrm{g_{m3}R_1})$$

2. Problem 4.22 in the course book.

KCL:

$$g_m(V_{in1} - V_x) + \frac{V_{out1} - V_x}{R_p} + \frac{V_{out1}}{R_D} = 0 \leftrightarrow V_x\left(g_m + \frac{1}{R_P}\right) = g_m V_{in1} + \left(\frac{1}{R_P} + \frac{1}{R_D}\right)V_{out1} \ (1)$$

$$g_m(V_{in2} - V_x) + \frac{V_{out2}}{R_D} = 0 \leftrightarrow V_{out2} = -g_m R_D V_{in} \ + g_m R_D V_x \ (2)$$

(1) in (2):

$$V_{out2} = -g_m R_D V_{in2} + \frac{g_m R_D}{g_m + \frac{1}{R_P}}\left(g_m V_{in1} + \left(\frac{1}{R_O} + \frac{1}{R_D}\right)V_{out1}\right) \leftrightarrow$$

$$(g_m R_P + 1)V_{out2} - (g_m R_P + g_m R_D)V_{out1} = g_m^2 R_P R_D (V_{in1} - V_{in2}) - g_m R_D V_{in2}$$

Let $V_{out1} + V_{out2} = 0$ and $V_{out} = V_{out1} - V_{out2} \leftrightarrow V_{out1} = \frac{V_{out}}{2}, V_{out2} = -\frac{V_{out}}{2}$ and $V_{in2} = \frac{V_{in1}+V_{in2}}{2} - \frac{V_{in1}-V_{in2}}{2}$

$$\left(g_m R_p + \frac{1}{2} + \frac{g_m R_D}{2}\right)V_{out} = -g_m R_D\left(g_m R_P + \frac{1}{2}\right)(V_{in1} - V_{in2}) + \frac{g_m R_D}{2}(V_{in1} + V_{in2}) \leftrightarrow$$

$$\left(\frac{1}{R_D} + \frac{1}{2R_P} + \frac{1}{2g_m R_P R_D}\right)V_{out} = -\left(g_m + \frac{1}{2R_P}\right)(V_{in1} - V_{in} \ ) + \frac{V_{in1} + V_{in2}}{2R_P}$$

3. Problem 5.3(a) in the course book for $\gamma = 0$. Assume all the transistors are in the saturation region and $\mu_n C_{ox} = 4\mu_p C_{ox} = 200\ \mu A/V^2$, $V_{tn} = |V_{tp}| = 0.5\ V$ and $V_{DD} = 2.5\ V$.

Calculate V_P:

$$V_{GS} = \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}\frac{W}{L}}} + V_{tn} = \sqrt{\frac{5I_{ref}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_N}} + V_{tn} \approx 0.85\ V$$

$$V_P = V_{in,CM} - V_{GS} \approx 1.3\ V - 0.85\ V = 0.45\ V$$

Calculate diode-connected PMOS's drain voltages

Mirror:

$$V_D = V_{DD} - |V_{GS1}| = V_{DD} - \left|\sqrt{\frac{4I_{ref}}{\mu_p C_{ox}\left(\frac{W}{L}\right)_P}} + V_{tp}\right| \approx 1.37\ V$$

Load:

$$V_D = V_{DD} - |V_{GS1}| = V_{DD} - \left|\sqrt{\frac{2I_D}{\mu_p C_{ox}\left(\frac{W}{L}\right)_P}} + V_{tp}\right| = V_{DD} - \left|\sqrt{\frac{2\left(\frac{5}{2}I_{ref} - 2I_{ref}\right)}{\mu_p C_{ox}\left(\frac{W}{L}\right)_P}} + V_{tp}\right| \approx 1.68\ V$$

4. Calculate the small-signal voltage gain of the cascade differential pair shown in Figure 3. Assume $\gamma = 0$.

We use the equivalent half-circuit since the circuit is symmetric.
The Norton equivalent amplifier is a current source in parallel with an output impedance, its gain is $\frac{V_{out}}{V_{in}} = G_m R_{out}$ where $G_m = \frac{I_{out}}{V_{in}}$
We find G_m by connecting V_{out} to ground and finding I_{out}.

$$I_{out} = -g_{m3}V_x - \frac{V_x}{r_{o3}} = -\left(g_{m3} + \frac{1}{r_{o3}}\right)V_x \leftrightarrow V_x = -\frac{I_{out}}{g_{m3} + \frac{1}{r_{o3}}} = -\left(\frac{1}{g_{m3}} \parallel r_{o3}\right)I_{out} \ (1)$$

$$I_{out} = g_{m1}V_{in} + \frac{V_x}{r_{o1}} \leftrightarrow V_x = (I_{out} - g_{m1}V_{in})r_{o1} \ (2)$$

(1) in (2):

$$(I_{out} - g_{m1}V_{in})r_{o1} = -\left(\frac{1}{g_{m3}} \parallel r_{o3}\right)I_{out} \leftrightarrow I_{out}\left(r_{o1} + \left(\frac{1}{g_{m3}} \parallel r_{o3}\right)\right) = g_{m1}r_{o1}V_{in} \leftrightarrow$$

$$G_m = \frac{I_{out}}{V_{in}} = \frac{g_{m1}r_{o1}}{r_{o1} + \frac{1}{g_{m3}} \parallel r_{o3}} \approx \frac{g_{m1}r_{o1}}{r_{o1}} = g_{m1}$$

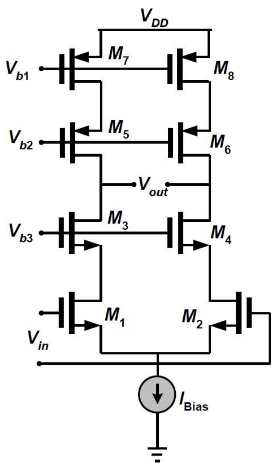

Figure 3 A differential amplifier.

5. Consider the differential amplifier shown in Figure 4. Due to a manufacturing defect, a large parasitic resistance has appeared between the drains of $M_1$and M_4. Assume $\lambda = \gamma = 0$. Calculate the small-signal gain, common-mode gain, and CMRR. Assume that $\left(W/L\right)_1 = \left(W/L\right)_2$ and $\left(W/L\right)_3 = \left(W/L\right)_4$.

First do an KCL for the ground nodes, remember that $g_{m1} = g_{m2}$ since $V_{GS1} = V_{GS}$ and $\frac{W}{L}_1 = \frac{W}{L}_2$:

$$\frac{V_{out1}}{R_D} + \frac{V_{out2}}{R_D} + g_{m1}\Delta V_{in} - g_{m1}\Delta V_{in} = 0 \leftrightarrow V_{out1} = -V_{out2}$$

Then do a KCL at V_{out1}:

$$\frac{V_{out1}}{R_D} - g_{m3}V_x = 0 \leftrightarrow V_x = \frac{V_{out1}}{g_{m3}R_D}$$

And a KCL at V_x:

$$g_{m3}V_x + g_{m1}\Delta V_{in} - \frac{V_{out2} - V_x}{R_P} = 0 \leftrightarrow V_{out2}\left(\frac{1}{R_P}\right) = V_x\left(g_{m3} + \frac{1}{R_P}\right) + g_{m1}\Delta V_{in} \leftrightarrow$$

$$V_{out2} = \frac{V_{out}\ R_P}{g_{m3}R_D}\left(g_{m3} + \frac{1}{R_P}\right) + g_{m1}R_P\Delta V_{in} \leftrightarrow V_{out1}\left(1 + \frac{R_P}{R_D} + \frac{1}{g_{m3}R_D}\right) = -g_{m1}R_P\Delta V_{in} \leftrightarrow$$

$$\frac{V_{out1}}{\Delta V_{in}} = -\frac{g_{m1}R_P}{1 + \frac{R_P}{R_D} + \frac{1}{g_{m3}R_D}}$$

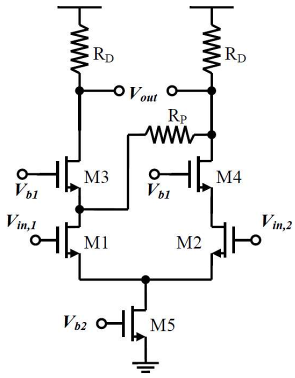

Figure 4 A differential amplifier.

6. A simple current mirror is shown in Figure 5. Calculate the value of V_{bias} in order to have $V_N = {}^{V_{DD}}/_{2}$. Using this value calculate the error percentage of mirroring. Error percentage can be defined as $E(\%) = \frac{|I_{ref} - I_{out}|}{I_{ref}} \times 100$.

$$\mu_n C_{ox} = 200\frac{\mu A}{V^2}$$

$$\mu_p C_{ox} = 50\frac{\mu A}{V^2}$$

$$V_{DD} = 3\ V$$

$$\left(\frac{W}{L}\right)_3 = 4\left(\frac{W}{L}\right)_1 = 4\left(\frac{W}{L}\right)_2 = 20$$

$$R = 1\ k\Omega$$

$$V_{tn} = |V_{tp}| = 0.5\ V$$

$$\lambda_n = |\lambda_p| = 0.1\ V^{-1}$$

Compute V_{bias} so that $V_N = \frac{V_{DD}}{2}$. Assume saturation.

M3: $I_{ref} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_3 \left(V_{DD} - V_{bias} - |V_{tp}|\right)^2 \left(1 + |\lambda_p|(V_{DD} - V_N)\right)$ (1)

M1: $I_{ref} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_N - V_{tn})^2(1 + \lambda_n V_N)$ (2)

Divide (1) by (2):

$$1 = \frac{\mu_p C_{ox}\left(\frac{W}{L}\right)_3}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1} \frac{\left(V_{DD} - V_{bias} - |V_{tp}|\right)^2}{(V_N - V_{tn})^2} \frac{\left(1 + |\lambda_p|(V_{DD} - V_N)\right)}{1 + \lambda_n V_N} \leftrightarrow$$

$$1 = \left(\frac{V_{DD} - V_{bias} - |V_{tp}|}{V_N - V_{tn}}\right)^2 \leftrightarrow V_{bias} = V_{DD} - V_N + V_{tn} - |V_{tp}| = V_{DD} - V_N = \frac{V_{DD}}{2}$$

Compute the error percentage $E(\%) = \frac{|I_{ref} - I_{out}|}{I_{ref}}$

$$I_{out} = \frac{1}{2}\mu_n C_{px}\left(\frac{W}{L}\right)_2 (V_N - V_{tn})^2\left(1 + \lambda_n(V_{DD} - I_{out}R)\right) \leftrightarrow$$

$$I_{out}\left[1 + \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_N - V_{tn})^2 \lambda_n R\right] = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_N - V_{tn})^2(1 + \lambda_n V_{DD}) \leftrightarrow$$

$$I_{out} = \frac{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_N - V_{tn})^2(1 + \lambda_n V_{DD})}{1 + \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_N - V_{tn})^2 \lambda_n R}$$

$$\frac{I_{out}}{I_{ref}} = \frac{1 + \lambda_n V_{DD}}{1 + \lambda_n V_N} \frac{1}{1 + \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_N - V_{tn})^2 \lambda_n R} \approx 1.0766$$

$E(\%) \approx 7.66\ \%$

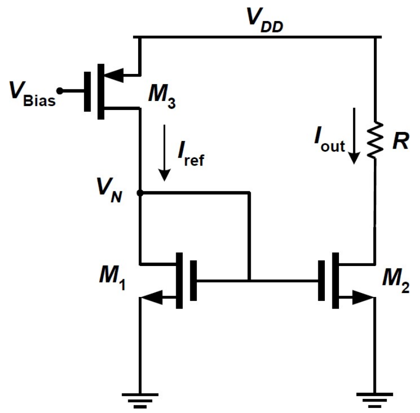

Figure 5 Simple current mirror.