Exercises for Tutorial 1: Single Ended Amplifiers

1. Problem 3.20 in the course book

a) $KCL\ at\ v_{out}$:

$$\frac{v_{out} - v_{in}}{R_F} + g_{m1} v_{gs1} + \frac{v_{out}}{r_{o1} \parallel R_D} = 0 \Rightarrow / v_{gs1} = v_{in} / \Rightarrow$$

$$v_{out}\left(\frac{1}{R_F} + \frac{1}{r_{o1} \parallel R_D}\right) = -v_{in}\left(g_{m1} - \frac{1}{R_F}\right) \Rightarrow$$

$$A_V = \frac{v_{out}}{v_{in}} = -\frac{g_{m1} - \frac{1}{R_F}}{\frac{1}{R_F} + \frac{1}{r_{o1} \parallel R_D}}$$

b) $KCL\ at\ v_{out}$:

$$g_{m1} v_{gs1} + \frac{v_{out} - v_{in}}{r_{o1} \parallel R_1} + \frac{v_{out}}{R_2} = 0 \Rightarrow / v_{gs1} = -v_{in} / \Rightarrow$$

$$v_{out}\left(\frac{1}{r_{o1} \parallel R_1} + \frac{1}{R_2}\right) = v_{in}\left(g_{m1} + \frac{1}{r_{o1} \parallel R_1}\right) \Rightarrow$$

$$A_V = \frac{v_{out}}{v_{in}} = \frac{g_{m1} + \frac{1}{r_{o1} \parallel R_1}}{\frac{1}{r_{o1} \parallel R_1} + \frac{1}{R_2}}$$

c) $KCL\ at\ v_{out}$:

$$g_{m1} v_{gs1} - g_{m2} v_{gs2} + \frac{v_{out}}{r_{o1} \parallel r_{o2}} = 0 \Rightarrow / \begin{matrix} v_{gs1} = v_{in} \\ v_{gs2} = v_{in} - v_{out} \end{matrix} / \Rightarrow$$

$$v_{out}\left(\frac{1}{r_{o1} \parallel r_{o2}} + g_{m2}\right) = -v_{in}(g_{m1} - g_{m2}) \Rightarrow$$

$$A_V = \frac{v_{out}}{v_{in}} = -\frac{g_{m1} - g_{m2}}{g_{m2} + \frac{1}{r_{o1} \parallel r_{o2}}}$$

d) $KCL\ at\ v_x\ and\ v_{out}.$

$$v_x\colon g_{m1}v_{in}+\frac{v_x}{r_{o1}}+\frac{v_x-v_{out}}{R_D}=0\Leftrightarrow v_x\left(\frac{1}{r_{o1}}+\frac{1}{R_D}\right)=-g_{m1}v_{in}+\frac{v_{out}}{R_D}\ (1)$$

$$v_{out}\colon \frac{v_{out}-v_x}{R_D}+g_{m2}v_x+\frac{v_{out}}{r_{o2}}=0\Leftrightarrow v_{out}\left(\frac{1}{R_D}+\frac{1}{r_{o2}}\right)=v_x\left(-g_{m2}+\frac{1}{R_D}\right)\ (2)$$

Insert (1) into (2):

$$v_{out}\left(\frac{1}{R_D}+\frac{1}{r_{o2}}\right)\left(\frac{1}{r_{o1}}+\frac{1}{R_D}\right)=\left(-g_{m1}v_{in}+\frac{v_{out}}{R_D}\right)\left(-g_{m2}+\frac{1}{R_D}\right)=$$

$$g_{m1}g_{m2}v_{in}-\frac{g_{m1}}{R_D}v_{in}-\frac{g_{m2}}{R_D}v_{out}+\frac{v_{out}}{R_D^2}\Leftrightarrow$$

$$v_{out}\left(\frac{1}{R_Dr_{o1}}+\frac{1}{R_D^2}+\frac{1}{r_{o2}r_{o1}}+\frac{1}{r_{o2}R_D}-\frac{1}{R_D^2}+\frac{g_{m2}}{R_D}\right)=v_{in}\left(g_{m1}g_{m2}-\frac{g_{m1}}{R_D}\right)\Leftrightarrow$$

$$v_{out}\left(\frac{1}{r_{o1}}+\frac{R_D}{r_{o1}r_{o2}}+\frac{1}{r_{o2}}+g_{m2}\right)=v_{in}(g_{m1}g_{m2}R_D-g_{m1})\Leftrightarrow$$

$$A_V=\frac{v_{out}}{v_{in}}=\frac{g_{m1}(g_{m2}R_D-1)}{\frac{1}{r_{o1}}+g_{m2}+\frac{1}{r_{o2}}\left(1+\frac{R_D}{r_{o1}}\right)}$$

e) $KCL\ at\ v_x\ and\ v_{out}.$

$$(v_x): \frac{v_x}{R_S} - g_{m1}(v_{in} - v_x) + \frac{v_x - v_{out}}{r_{o1}} = 0 \Leftrightarrow v_x\left(g_{m1} + \frac{1}{R_S} + \frac{1}{r_{o1}}\right) = g_{m1}v_{in} + \frac{v_{out}}{r_{o1}}\ (1)$$

$$(v_{out}): g_{m1}(v_{in} - v_x) + g_{m2}v_x + \frac{v_{out} - v_x}{r_{o1}} + \frac{v_{out}}{r_{o2}} = 0 \Leftrightarrow$$

$$v_{out}\left(\frac{1}{r_{o1}} + \frac{1}{r_{o2}}\right) = v_x\left(g_{m1} - g_{m2} + \frac{1}{r_{o1}}\right) - g_{m1}v_{in}\ (2)$$

Insert (1) into (2):

$$v_{out}\left(\frac{1}{r_{o1}} + \frac{1}{r_{o2}}\right) = \frac{\left(g_{m1}v_{in} + \frac{v_{out}}{r_{o1}}\right)\left(g_{m1} - g_{m2} + \frac{1}{r_{o1}}\right)}{g_{m1} + \frac{1}{R_S} + \frac{1}{r_{o1}}} - g_{m1}v_{in} \Leftrightarrow$$

$$v_{out}\left(\frac{1}{r_{o1}} + \frac{1}{r_{o2}}\right)\left(g_{m1} + \frac{1}{R_S} + \frac{1}{r_{o1}}\right)$$

$$= \left(g_{m1}v_{in} + \frac{v_{out}}{r_{o1}}\right)\left(g_{m1} - g_{m2} + \frac{1}{r_{o1}}\right) - g_{m1}v_{in}\left(g_{m1} + \frac{1}{R_S} + \frac{1}{r_{o1}}\right) \Leftrightarrow$$

$$v_{out}\left(\frac{1}{r_{o1}} + \frac{1}{r_{o2}}\right)\left(g_{m1} + \frac{1}{R_S} + \frac{1}{r_{o1}}\right) = -v_{in}g_{m1}\left(g_{m2} + \frac{1}{R_S}\right) + \frac{v_{out}}{r_{o1}}\left(g_{m1} - g_{m2} + \frac{1}{r_{o1}}\right) \Leftrightarrow$$

$$v_{out}\left(\frac{g_{m1}}{r_{o1}} + \frac{1}{r_{o1}R_S} + \frac{1}{r_{o1}^2} + \frac{g_{m1}}{r_{o2}} + \frac{1}{r_{o2}R_S} + \frac{1}{r_{o1}r_{o2}} - \frac{g_{m1}}{r_{o1}} + \frac{g_{m2}}{r_{o1}} - \frac{1}{r_{o1}^2}\right) = -v_{in}g_{m1}\left(g_{m2} + \frac{1}{R_S}\right) \Leftrightarrow$$

$$v_{out}\left(g_{m1} + \frac{1}{R_S} + \frac{1}{r_{o1}}\left(1 + r_{o2}\left(g_{m2} + \frac{1}{R_S}\right)\right)\right) = -v_{in}g_{m1}\left(g_{m2} + \frac{1}{R_S}\right)r_{o2} \Leftrightarrow$$

$$A_V = \frac{v_{out}}{v_{in}} = -\frac{g_{m1}\left(g_{m2} + \frac{1}{R_S}\right)r_{o2}}{g_{m1} + \frac{1}{R_S} + \frac{1}{r_{o1}}\left(1 + r_{o2}\left(g_{m2} + \frac{1}{R_S}\right)\right)}$$

2. Problem 3.21(h) in the course book

$KCL\ at\ v_x\ and\ v_{out}:$

$$(v_x): g_{m1}v_{in} + g_{m3}v_{out} + \frac{v_x}{r_{o1}} + \frac{v_x}{r_{o3}} = 0 \Leftrightarrow v_x = -(g_{m1}v_{in} + g_{m3}v_{out})r_{o1} \parallel r_{o3}\ (1)$$

$$(v_{out}): -g_{m2}(v_x - v_{out}) + \frac{v_{out}}{r_{o2}} = 0 \Leftrightarrow v_{out}\left(g_{m2} + \frac{1}{r_{o2}}\right) = g_{m2}v_x\ (2)$$

$$(1)\ in\ (2): v_{out}\left(g_{m2} + \frac{1}{r_{o2}}\right) = -g_{m1}g_{m2}(r_{o1} \parallel r_{o3})v_{in} - g_{m2}g_{m3}(r_{o1} \parallel r_{o3})v_{out} \Leftrightarrow$$

$$v_{out}\left(g_{m2}\left(1 + \frac{g_{m3}}{\frac{1}{r_{o1}} + \frac{1}{r_{o3}}}\right) + \frac{1}{r_{o2}}\right) = -v_{in}\frac{g_{m1}g_{m2}}{\frac{1}{r_{o1}} + \frac{1}{r_{o3}}} \Leftrightarrow$$

$$v_{out}\left(g_{m2}\left(\frac{1}{r_{o1}} + \frac{1}{r_{o3}} + g_{m3}\right) + \frac{1}{r_{o1}r_{o2}} + \frac{1}{r_{o2}r_{o3}}\right) = -v_{in}g_{m1}g_{m2} \Leftrightarrow$$

$$A_V = \frac{v_{out}}{v_{in}} = -\frac{g_{m1}g_{m2}}{g_{m2}\left(\frac{1}{r_{o1}} + \frac{1}{r_{o3}} + g_{m3}\right) + \frac{1}{r_{o1}r_{o2}} + \frac{1}{r_{o2}r_{o3}}}$$

$$= -\frac{g_{m1}g_{m2}r_{o1}r_{o2}r_{o3}}{g_{m2}g_{m3}r_{o1}r_{o2}r_{o3} + g_{m2}r_{o2}r_{o3} + g_{m2}r_{o1}r_{o2} + r_{o3} + r_{o11}}$$

$$= -\frac{g_{m1}g_{m2}r_{o1}r_{o2}r_{o3}}{(r_{o1} + r_{o3})(1 + g_{m2}r_{o2}) + g_{m2}g_{m3}r_{o1}r_{o2}r_{o3}}$$

3. Problem 3.27 in the course book. Assume $\mu_n C_{ox} = 200\ \mu A/V^2$, $V_{t0,n} = 0.5\ V$, $|2\Phi_f| = 0.9\ V$ and $V_{DD} = 3\ V$. Also assume that in part (a) both transistors are in saturation region.

a) $\lambda = \gamma = 0$

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\frac{W_1}{L_1}\left(V_{GS1} - V_{t0,n}\right)^2 = 0.5\ mA \Leftrightarrow \frac{W_1}{L_1} = \frac{2 \times 0.5\ mA}{\mu_n C_{ox}\left(V_{GS1} - V_{t0,n}\right)^2}$$

$$\frac{W_2}{L_2} = \frac{2 \times 0.5\ mA}{\mu_n C_{ox}\left(V_{GS2} - V_{t0,n}\right)^2}$$

$$V_{GS1} = V_{in} - V_{out} = 1\ V \text{ and } V_{GS2} = V_{GS1} = 0.5\ V \Rightarrow V_{GS2} = 1.5\ V$$

$$\frac{W_1}{L_1} = \frac{2 \times 0.5 \times 10^{-3}}{200 \times 10^{-6}(1 - 0.5)^2} = 20$$

$$\frac{W_2}{L_2} = \frac{2 \times 0.5 \times 10^{-3}}{200 \times 10^{-6}(1.5 - 0.5)^2} = 5$$

b) $\gamma = 0.45\ {}^1/_V$ and $V_{in} = 2.5\ V \Leftrightarrow M_1$ has body effect, $V_{SB1} = V_{out} = V_{in} - 1\ V = 1.5\ V$

$$\frac{W_1}{L_1} = \frac{2I_{D1}}{\mu_n C_{ox}(V_{GS1} - V_{t1})^2},\ V_{t1} = V_{t0,n} + \gamma\left(\sqrt{|2\Phi_f + V_{SB1}|} - \sqrt{2\Phi_f}\right) = 0.77\ V$$

$$\frac{W_2}{L_2} = \frac{2I_{D2}}{\mu_n C_{ox}\left(V_{GS2} - V_{t0,n}\right)^2} = 5\ (Same\ as\ in\ a)$$

$$\frac{W_1}{L_1} = \frac{2 \times 0.5 \times 10^{-3}}{200 \times 10^{-6}(1 - 0.77\ )^2} = 94.5$$

M_2 is at the edge of saturation when $V_{out} = V_{GS2} - V_{t0,n} = 1.5\ V - 0.5\ V = 1\ V$

M_2 keeps current $0.5\ mA$ so:

$$\begin{cases} V_{t1} = V_{t0,n} + \gamma\left(\sqrt{|2\Phi_f + V_{out}|} - \sqrt{|2\Phi_f|}\right) = 0.69\ V \\ V_{in} = V_{out} + V_{GS1} = V_{out} + \sqrt{\dfrac{2I_{D1}}{\mu_n C_{ox}\frac{W_1}{L_1}}} + V_{t1} = 1 + 0.23 + 0.69 = 1.92\ V \end{cases}$$

$$V_{in,min} = 1.92\ V$$

4. A two-stage single-ended amplifier is shown in Figure 1. Calculate the small-signal voltage gain and the output resistance. Assume $g_m \gg 1/r_{o1}$ and $R_D \ll r_{o2}$. ($\lambda \neq 0$ and $\gamma = 0$).

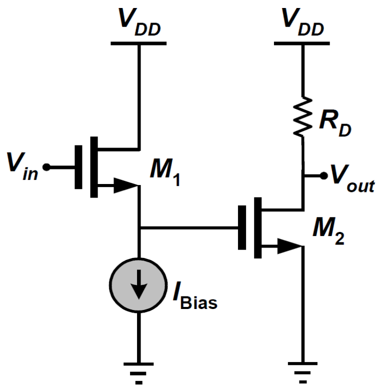

Figure 1 A two-stage amplifier

$KCL\ at\ v_x\ and\ v_{out}$:

$$(v_x): \frac{v_x}{r_{o1}} = g_{m1}(v_{in} - v_x) \Leftrightarrow v_x\left(g_{m1} + \frac{1}{r_{o1}}\right) = g_{m1}v_{in} \Leftrightarrow v_x = v_{in}\frac{g_{m1}}{g_{m1} + \frac{1}{r_{o1}}}\ (1)$$

$$(v_{out}): g_{m2}v_x + \frac{v_{out}}{r_{o2}} + \frac{v_{out}}{R_D} = 0 \Leftrightarrow v_{out}\left(\frac{1}{r_{o2}} + \frac{1}{R_D}\right) = -v_x g_{m2}\ (2)$$

$(1) in\ (2)$:

$$v_{out}\left(\frac{1}{r_{o2}} + \frac{1}{R_D}\right) = -v_{in}\frac{g_{m1}g_{m2}}{g_{m1} + \frac{1}{r_{o1}}} \Leftrightarrow$$

$$A_V = \frac{v_{out}}{v_{in}} = -\frac{g_{m1}g_{m2}}{\left(g_{m1} + \frac{1}{r_{o1}}\right)\left(\frac{1}{r_{o2}} + \frac{1}{R_D}\right)} \approx -g_{m2}R_D$$

$$R_{out} = \frac{V_Z}{I_Z} = \frac{I_Z(r_{o2} \parallel R_D)}{I_Z} = r_{o2} \parallel R_D \approx R_D \quad (R_D \ll r_{o2})$$

5. Using the small-signal model, calculate the voltage gain of the cascade stage shown in Figure 2 For both transistors we assume $g_m \gg g_{mb}$ and $r_o \to \infty$.

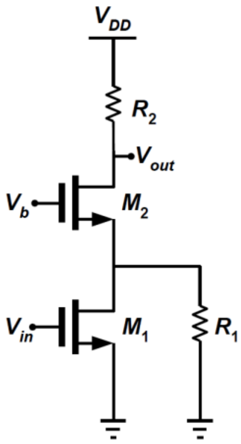

Figure 2 A cascode amplifier stage

$$g_{m1}v_{in} + g_{m2}v_x + \frac{v_x}{R_1} = 0 \Leftrightarrow v_x = -v_{in}\frac{g_{m1}}{g_{m2} + \frac{1}{R_1}} \ (1)$$

$$-g_{m2}v_x + \frac{v_{out}}{R_2} = 0 \Leftrightarrow v_{out} = v_x g_{m2} R_2 \ (2)$$

$$(1) and\ (2): A_V = \frac{v_{out}}{v_{in}} = -\frac{g_{m1}g_{m2}R_2}{g_{m2} + \frac{1}{R_1}}$$