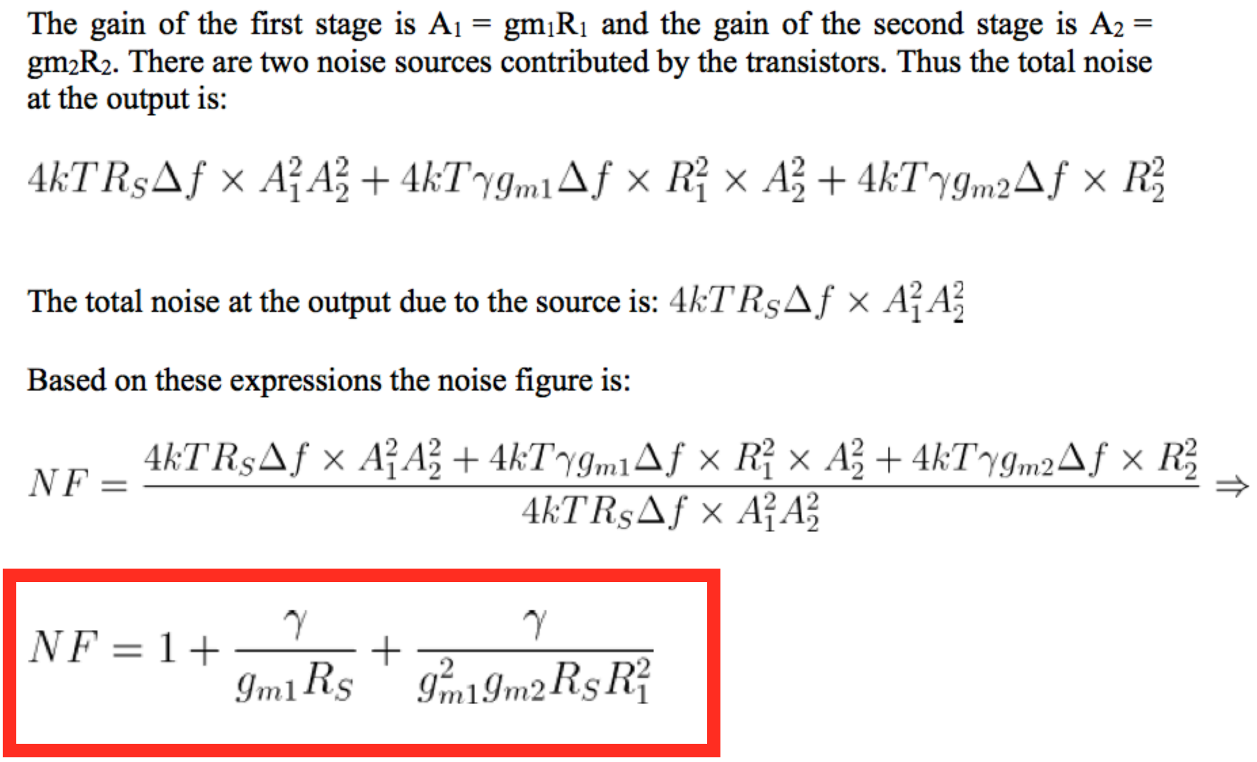
1.

See Tutorial 1, problem 3:



The gain of the first stage is $A_1 = g_{m1}R_1$ and the gain of the second stage is $A_2 = g_{m2}R_2$. There are two noise sources contributed by the transistors. Thus the total noise at the output is:

$$4kTR_S\Delta f \times A_1^2A_2^2 + 4kT\gamma g_{m1}\Delta f \times R_1^2 \times A_2^2 + 4kT\gamma g_{m2}\Delta f \times R_2^2$$

The total noise at the output due to the source is: $4kTR_S\Delta f \times A_1^2A_2^2$

Based on these expressions the noise figure is:

$$NF = \frac{4kTR_S\Delta f \times A_1^2A_2^2 + 4kT\gamma g_{m1}\Delta f \times R_1^2 \times A_2^2 + 4kT\gamma g_{m2}\Delta f \times R_2^2}{4kTR_S\Delta f \times A_1^2A_2^2} \Rightarrow$$

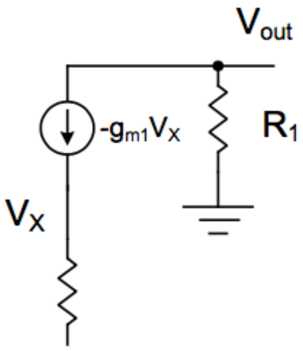
$$\boxed{NF = 1 + \frac{\gamma}{g_{m1}R_S} + \frac{\gamma}{g_{m1}^2 g_{m2} R_S R_1^2}}$$

(Noise equations with Δf are used above. Razavi and we in TSEK03 just assume Δf = 1 Hz, so just replace Δf with 1’s above!)

2.

As a start, see Tutorial 6, problem 2. But here not matched, gm ≠ R_S. This is actually Problem 5.8 from the book.

1. Calculate the votlage gain



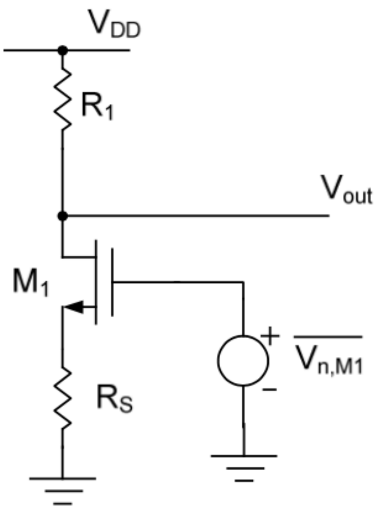
$$V_{out} = g_{m1}V_X R_1 \quad \rightarrow \quad A_X = \frac{V_{out}}{V_X} = g_{m1}R_1$$

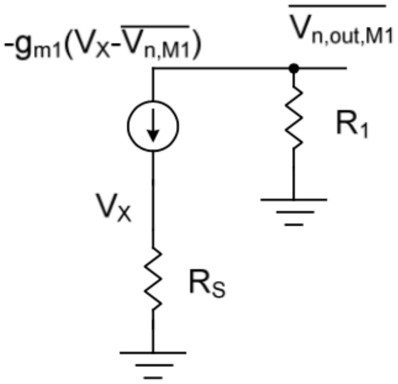
Since $R_{in} = \dfrac{1}{g_{m1}}$, we have the voltage gain from Vin to Vout:

$$A_V = \frac{\frac{1}{g_{m1}}}{R_S + \frac{1}{g_{m1}}} A_X = \frac{g_{m1}R_1}{1 + R_S g_{m1}}$$

2. Calculate noise figure
   The noise contributions are from M1, R1, and Rs.

   a. Calculate output noise from M1





2. (continued)

$A_{M1} = -\frac{R_1}{R_S + \frac{1}{g_{m1}}}$, hence the output noise of M1 can be obtained from

$$\overline{V^2_{n,out}}\Big|_{M1} = \frac{4kT\gamma}{g_{m1}} A^2_{M1} = \frac{4kT\gamma}{g_{m1}} \left(\frac{R_1}{R_S + \frac{1}{g_{m1}}}\right)^2$$

b. Calculate output noise from R1

$$\overline{V^2_{n,out}}\Big|_{R_1} = 4kTR_1$$

The noise of R_S is multiplied by the gain when referred to the output, and the result is divided by the gain when referred to the input.

We thus have:

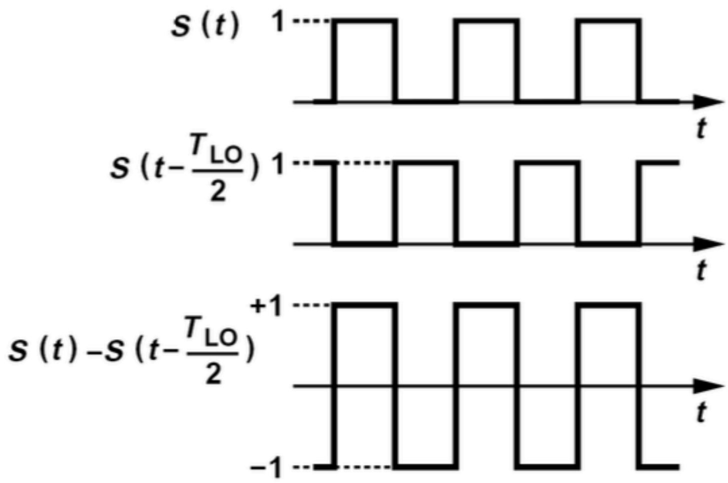
$$NF = \frac{1}{4kTR_S} \frac{\overline{V^2_{n,out}}}{A^2_V}$$

$$= 1 + \frac{4kTR_1}{4kTR_S \left(\frac{g_{m1}R_1}{1+R_S g_{m1}}\right)^2} + \frac{\frac{4kT\gamma}{g_{m1}} \left(\frac{R_1}{R_S + \frac{1}{g_{m1}}}\right)^2}{4kTR_S \left(\frac{g_{m1}R_1}{1+R_S g_{m1}}\right)^2}$$

$$\boxed{= 1 + \frac{(1+R_S g_{m1})^2}{R_S R_1 g^2_{m1}} + \frac{\gamma}{g_{m1} R_S}}$$

3.

(a)



$$V_{out}(t) = I_{RF}R\left[S(t) - S(t - T_{LO}/2)\right]$$

If $V_{RF} = A_{RF}\cos(\omega_{RF}t)$, then by ignoring the higher order terms:

$$V_{IF} = V_{out} = \frac{4}{\pi}g_{m3}RA_{RF}cos(\omega_{RF}t)cos(\omega_{LO}t) = \frac{2}{\pi}g_{m3}RA_{RF}cos((\omega_{RF}-\omega_{LO})t)$$

Therefore the conversion gain is:

$$G_C = \frac{V_{IF}}{A_{RF}} = \frac{2}{\pi}g_{m3}R$$

(b)
$$\overline{V^2_{n,out,M_3}} = 4kT\gamma g_m R^2$$
$$\overline{V^2_{n,out,R_S}} = 4kTR_S(g_mR)^2$$
$$\overline{V^2_{n,out,R}} = 2 \times 4kTR$$

The noise figure can be written as:

$$NF = \frac{\overline{V^2_{n,out,M_3}} + \overline{V^2_{n,out,R_s}} + \overline{V^2_{n,out,R}}}{G_C^2\overline{V^2_{n,out,R_S}}} \boxed{= \frac{\pi^2}{4}\left(1 + \frac{\gamma}{g_mR_S} + \frac{2}{g_m^2R_SR}\right)}$$

4.

(a) DC phase shift = 180° (frequency $\rightarrow 0$)

Open loop circuit contains one pole

=> Frequency dependent phase shift = 90° (at frequency $\rightarrow \infty$)

Total phase shift = 90° + 180° = 270° => No oscillation.

(b) DC phase shift = 0° (frequency $\rightarrow 0$)

Open loop circuit contains two poles

=> Frequency dependent phase shift = 90° + 90° = 180° (at frequency $\rightarrow \infty$)

Total phase shift = 0° + 180° = 180° => No oscillation.

(c) DC phase shift = 180° (frequency $\rightarrow 0$)

Open loop circuit contains two poles

=> Frequency dependent phase shift = 90° + 90° = 180° (at frequency $\rightarrow \infty$)

Total phase shift = 180° + 180° = 360°

But as frequency $\rightarrow \infty$ the loop gain vanishes => No oscillation.

(d) DC phase shift = 180° (frequency $\rightarrow 0$)

Open loop circuit contains three poles

=> Frequency dependent phase shift = 90° + 90° + 90° = 270° (at frequency $\rightarrow \infty$)

Total phase shift = 180° + 270° = 450° or 90°

However at $f = f_{p1,2,3} < \infty$, where f_p is the cut-off frequency of a stage, the frequency dependent phase shift = 45° + 45° + 45° = 135°

Therefore for some $f_p < f < \infty$ the circuit may oscillate with a sufficient loop gain.

5.

a. $$H(s) = \frac{\phi_{out}}{\phi_{in}}(s) = \frac{K_{PD}K_{VCO}}{R_1C_1s^2 + s + K_{PD}K_{VCO}}.$$

b. Type-I PLL, only one pole.

c. For slow variations ($s \approx 0$) => $H(s) = 1$, and the output phase tracks the input.

6.

a. B Class-A has 50 % maximum theoretical efficiency, class-B 78.5 %.

b. D Class-D has 100 % maximum theoretical efficiency.

c. A Class-A has the best linearity among the linear classes (A, AB, B, C).

d. A Class-D is a switching amplifier (on/off), it has very bad linearity.