1.

The gain of the first stage is $A_1 = gm_1R_1$ and the gain of the second stage is $A_2 = gm_2R_2$. There are two noise sources contributed by the transistors. Thus the total noise at the output is:

$$4kTR_S\Delta f \times A_1^2A_2^2 + 4kT\gamma g_{m1}\Delta f \times R_1^2 \times A_2^2 + 4kT\gamma g_{m2}\Delta f \times R_2^2$$

The total noise at the output due to the source is: $4kTR_S\Delta f \times A_1^2A_2^2$

Based on these expressions the noise figure is:

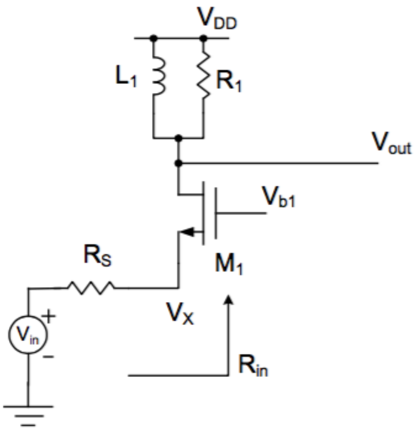
$$NF = \frac{4kTR_S\Delta f \times A_1^2A_2^2 + 4kT\gamma g_{m1}\Delta f \times R_1^2 \times A_2^2 + 4kT\gamma g_{m2}\Delta f \times R_2^2}{4kTR_S\Delta f \times A_1^2A_2^2} \Rightarrow$$

$$NF = 1 + \frac{\gamma}{g_{m1}R_S} + \frac{\gamma}{g_{m1}^2 g_{m2}R_SR_1^2}$$

2.

a. See Example 5.3 in Razavi. New value for Q = 2450/(2550-2350) = 12.25 ≈ 12.
b. On-chip L:s are usually much more lossy than on-chip C:s.
c. In the L, the Q value is mainly limited by the series resistance in the metallization and, at higher frequencies, the losses in the substrate.

3. Start by calculating R_{in} and A_V:



$$R_{in} = \frac{1}{g_{m1}} \text{ and } A_X = g_{m1}R_1$$

Assume input matching, $R_S = \frac{1}{g_{m1}}$ and $A_V = \frac{R_1}{2R_S}$

Now we calculate the noise figure. The noise contributions are from M1, M2, R1, and Rs.

a. The output noise power of M1.
   Similar to Problem 5.8 (or refer to the book on page 277)

$$\overline{V_{n,out}^2}\Big|_{M1} = \frac{4kT\gamma}{g_{m1}}\left(\frac{R_1}{R_S + \frac{1}{g_{m1}}}\right)^2 = kT\gamma\frac{R_1^2}{R_S}$$

b. Calculate the output noise power from R1

$$\overline{V_{n,out}^2}\Big|_{R_1} = 4kTR_1$$

c. Calculate the output noise power from M2
   First, we calculate the noise voltage gain from M2 to output.

$$A_{M2} = -\frac{g_{m2}R_1}{2}$$

Thus, the output noise power of M2:

$$\overline{V_{n,out}^2}\Big|_{M2} = \frac{4kT\gamma}{g_{m2}}\left(\frac{g_{m2}R_1}{2}\right)^2 = kT\gamma g_{m2}R_1^2$$

The noise of Rs is multiplied by the gain when referred to the output, and the result is divided by the gain when referred to the input. We thus have:

$$NF = \frac{1}{4kTR_S}\frac{\overline{V_{n,out}^2}}{A_V^2}$$

$$= 1 + \frac{4kTR_1}{4kTR_S\left(\frac{R_1}{2R_S}\right)^2} + \frac{kT\gamma\frac{R_1^2}{R_S}}{4kTR_S\left(\frac{R_1}{2R_S}\right)^2} + \frac{kT\gamma g_{m2}R_1^2}{4kTR_S\left(\frac{R_1}{2R_S}\right)^2}$$

$$= 1 + 4\frac{R_S}{R_1} + \gamma + \gamma g_{m2}R_S$$

4.

$$\left(\phi_{in}(s) - \frac{\phi_{out}(s)}{M}\right) \cdot K_{PD} \cdot \frac{1}{sRC+1} \cdot \frac{K_{VCO}}{s} = \phi_{out}(s)$$

$$\phi_{in}(s)\left(K_{PD} \cdot \frac{1}{sRC+1} \cdot \frac{K_{VCO}}{s}\right) = \phi_{out}(s)\left(1 + \frac{K_{PD}K_{VCO}}{s(sRC+1)M}\right)$$

$$\frac{\phi_{out}(s)}{\phi_{in}(s)} = \frac{\frac{K_{PD}K_{VCO}}{s(sRC+1)}}{1+\frac{K_{PD}K_{VCO}}{s(sRC+1)M}} = \frac{\frac{K_{PD}K_{VCO}}{s(sRC+1)}}{\frac{s(sRC+1)M + K_{PD}K_{VCO}}{s(sRC+1)M}} = \frac{K_{PD}K_{VCO}M}{s(sRC+1)M + K_{PD}K_{VCO}} = \frac{K_{PD}K_{VCO}M}{s^2RCM + sM + K_{PD}K_{VCO}}$$

$$\boxed{\frac{\phi_{out}(s)}{\phi_{in}(s)}} = \frac{\frac{K_{PD}K_{VCO}M}{RCM}}{s^2 + s\frac{M}{RCM} + \frac{K_{PD}K_{VCO}}{RCM}} = \boxed{\frac{\frac{K_{PD}K_{VCO}}{RC}}{s^2 + s\frac{1}{RC} + \frac{K_{PD}K_{VCO}}{RCM}}}$$

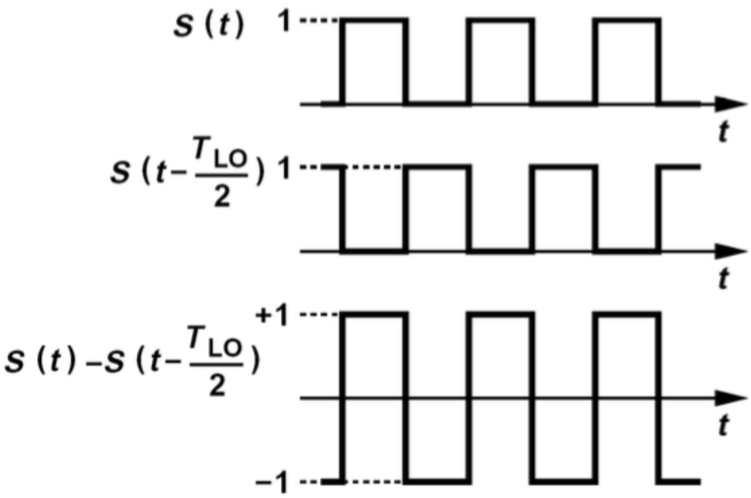
$s^2 + 2\xi\omega_n s + \omega_n^2$ =>

$\boxed{\omega_n = \sqrt{\frac{K_{PD}K_{VCO}}{RCM}}}$ and

$$2\xi\omega_n = \frac{1}{RC} \Rightarrow \boxed{\xi =} \frac{1}{2\omega_n RC} = \frac{1}{2RC\sqrt{\frac{K_{PD}K_{VCO}}{RCM}}} \boxed{= \frac{1}{2\sqrt{\frac{K_{PD}K_{VCO}RC}{M}}}}$$

5.

(a)



$$V_{out}(t) = I_{RF}R\left[S(t) - S(t - T_{LO}/2)\right]$$

If $V_{RF} = A_{RF}\cos(\omega_{RF}t)$, then by ignoring the higher order terms:

$$V_{IF} = V_{out} = \frac{4}{\pi}g_{m3}RA_{RF}cos(\omega_{RF}t)cos(\omega_{LO}t) = \frac{2}{\pi}g_{m3}RA_{RF}cos((\omega_{RF} - \omega_{LO})t)$$

Therefore the conversion gain is:

$$G_C = \frac{V_{IF}}{A_{RF}} = \frac{2}{\pi}g_{m3}R$$

(b)
$$\overline{V^2_{n,out,M_3}} = 4kT\gamma g_m R^2$$
$$\overline{V^2_{n,out,R_S}} = 4kTR_S(g_m R)^2$$
$$\overline{V^2_{n,out,R}} = 2 \times 4kTR$$

The noise figure can be written as:

$$NF = \frac{\overline{V^2_{n,out,M_3}} + \overline{V^2_{n,out,R_s}} + \overline{V^2_{n,out,R}}}{G_C^2\overline{V^2_{n,out,R_S}}} = \frac{\pi^2}{4}\left(1 + \frac{\gamma}{g_m R_S} + \frac{2}{g_m^2 R_S R}\right)$$

6.

See Example 12.6 in Razavi for the details.

6a: losses in the balun = 0.3 mW
6b: losses in the balun = 300 mW		this is more unfavorable