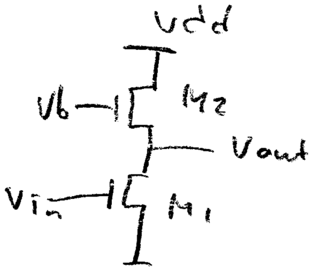
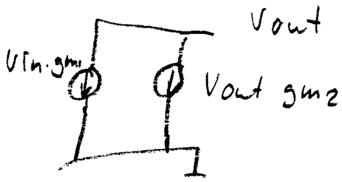
1.

Similar (but not identical) to Tutorial 6, problem 1, also similar (but again not identical) to Example 7.10 in Razavi's "Design of Analog CMOS Integrated Circuits", p. 226.

a.



⇒ SS:



$$V_{out} \cdot g_{m2} + V_{in} \cdot g_{m1} = 0$$

$$A_0 = \frac{V_{out}}{V_{in}} = -\frac{g_{m1}}{g_{m2}}$$

Noise-source independant; can be added

$$M_1: \overline{V^2_{ni,M1}} = \frac{4kT\gamma}{g_{m1}}$$ already at the input

$$M_2: \overline{V^2_{ni,M2}} = \frac{\overline{V^2_{no,M2}}}{(A_0)} = \frac{4kT\gamma}{g_{m2}} \cdot \left(\frac{g_{m2}}{g_{m1}}\right)^2 = \frac{4kT\gamma}{} \cdot \frac{g_{m2}}{(g_{m1})^2}$$

$$\overline{V^2_{ni,tot}} = \overline{V^2_{ni,M1}} + \overline{V^2_{ni,M2}} = \frac{4kT\gamma}{g_{m1}}\left(1 + \frac{g_{m2}}{g_{m1}}\right)$$

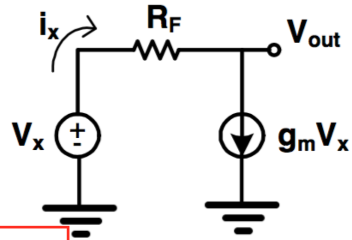
* compared to the tutorial and Razavi analog CMOS,

$\lambda = 0 \Rightarrow$ no CLM $\Rightarrow$ no r_{o1} & r_{o2}!

1 b. To minimize the (input-referred) noise, g_{m2} should be minimized. Since g_m is proportional to the square root of (W/L), the size of M_2 should be small (small W for fixed L).

2.

Small-signal model:



(a) $i_x = g_m V_x \rightarrow R_{in} = \frac{V_x}{i_x} = \frac{1}{g_m}$

(b) Gain from the gate of the transistor to the output:

$$\frac{V_{out} - V_x}{R_F} + g_m V_x = 0 \rightarrow \frac{V_{out}}{V_x} = 1 - g_m R_F$$

After matching: $g_m = 1/R_S \rightarrow A = \frac{V_{out}}{V_{in}} = \frac{1/g_m}{R_S + 1/g_m}(1 - g_m R_F) = \frac{1}{2}(1 - \frac{R_F}{R_S})$

If $R_F = 10R_S$, then A = -9/2 or -4.5

(c) The noise of R_S is multiplied by the gain square to appear at the output. Thus:

$$\overline{V^2_{n,out,R_S}} = 4kTR_S \cdot \frac{1}{4}(1 - \frac{R_F}{R_S})^2$$

If $R_F \gg R_S$:

$$\overline{V^2_{n,out,R_S}} \approx 4kTR_S \cdot \frac{1}{4}(-\frac{R_F}{R_S})^2 = kT\frac{R_F^2}{R_S}$$

3.

(a) Ignoring higher frequency terms:

$$i_{LO}(t) = \frac{1}{2} + \frac{2}{\pi}\cos\omega_{LO}(t) - \frac{2}{3\pi}\cos 3\omega_{LO}(t) + \ldots \quad \text{and} \quad i_{RF}(t) = I_{RF}\cos\omega_{RF}(t) + I_{BIAS}$$

Therefore in case of switching the output voltage at IF is given by:

$$i_{IF}(t) = \left[I_{RF}\cos\omega_{RF}(t) + I_{BIAS}\right] \times \left[\frac{1}{2} + \frac{2}{\pi}\cos\omega_{LO}(t)\right]$$

$$i_{IF}(t) = \frac{I_{RF}}{2}\cos\omega_{RF}(t) + \frac{I_{BIAS}}{2} + \frac{2I_{BIAS}}{\pi}\cos\omega_{LO}(t) + \frac{I_{RF}}{\pi}\cos(\omega_{LO} - \omega_{RF})t$$

Applying $V_{IF}(t) = -g_m R V_{RF}$, The conversion gain is given by:

$$G_C = \left|\frac{V_{IF}(t)}{V_{RF}(t)}\right| = \left|\frac{R \times i_{IF}(t)}{\frac{i_{RF}(t)}{g_m}}\right| = \frac{\frac{1}{\pi} g_m R I_{RF}}{I_{RF}} = \boxed{\frac{1}{\pi} g_m R}$$

similar to *gm* for our transistors, since long channel devices

(b)
$$\overline{V}^2_{nout,M3} = \overline{I}^2_{n,M3} R^2 = 4KT\gamma g_{do} R^2 \Delta f$$
$$\overline{V}^2_{nout,R_S} = \overline{V}^2_{n,R_S} A_V^2 = 4KTR_S (g_m R)^2 \Delta f$$
$$\overline{V}^2_{nout,R} = 4KTR\Delta f$$

The noise factor is given by:

$$F = \frac{\overline{V}^2_{nout,M3} + \overline{V}^2_{nout,R_S} + \overline{V}^2_{nout,R}}{(G_C)^2 \overline{V}^2_{n,R_S}} = \boxed{\pi^2\left(1 + \frac{\gamma}{g_m^2 R_S} + \frac{1}{g_m^2 R R_S}\right)}$$

The M_3 in this solution is called M_2 for our problem

4.

a. See Example 8.14 and Figure 8.26 in the book. Here instead we have Q=8 @ 2.45 GHz => Q*(L1+L2)*ω = 246 Ω.

g_m for the transistors > 246/2 = 123 Ω^{-1}.

b. See the book, pp. 511-512.

5.

Similar to Fig 9.30 in the course book and eq. 9.17 - 9.19.

The solution can also be written as:

Open loop transfer functions of the system is:

$$H_o(s) = K_{PFD} Z_{LPF}(s) \frac{K_{VCO}}{s} = \frac{I_o}{2\pi}\left(\frac{1}{sC_p} + R\right)\frac{K_{VCO}}{s}$$

$$H_o(s) = \frac{I_0 K_{VCO}}{2\pi C_p}\frac{1+sRC_p}{s^2} = k\frac{1+sRC_p}{s^2} \quad \text{, where } \boxed{k = \frac{I_0 K_{VCO}}{2\pi C_p}}$$

The close-loop transfer function is then:

$$\boxed{H(s) = \frac{H_0(s)}{1+H_0(s)} = k\frac{1+sRC_p}{s^2+sRC_pk+k}}$$

6.

a. B C E D A

b. The ”conduction angle” described how large portion of the total cycle an amplifier is conducting, see book, p. 780.