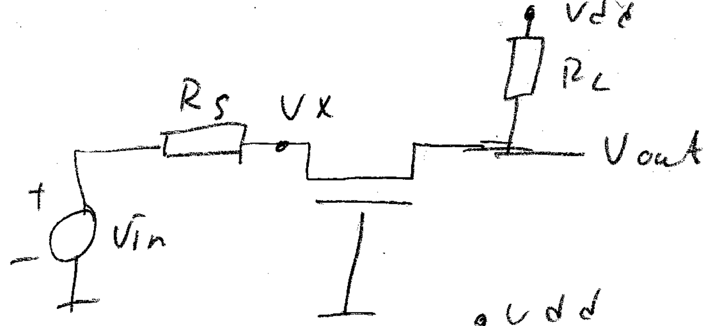
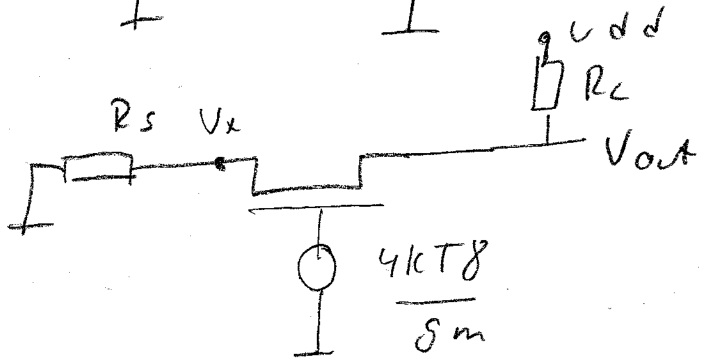
Solutions

1 a) Circuit :



For noise



i) Gain? $\frac{V_{out}}{V_x} = g_m \cdot R_L$

$$A_v = \frac{V_{out}}{V_{in}} = g_m \cdot R_L \left(\frac{\frac{1}{g_m}}{\frac{1}{g_m} + R_s} \right) = g_m \cdot R_L \left(\frac{1}{1 + g_m \cdot R_s} \right)$$

When used as an LNA, select $g_m = \frac{1}{R_s}$ ⇒

$$A_v = \frac{R_L}{2 R_s}$$

ii) Noise from M1: $\overline{V_{n,out\,M1}^2} = \frac{4kT\gamma}{g_m} \cdot \left(\frac{R_L}{R_s + \frac{1}{g_m}} \right)^2 = kT\gamma \frac{R_L^2}{R_s}$

Noise from R_L = $\overline{V_{n,out,RL}^2} = 4kT R_L$

iii) NF: $1 + \frac{\overline{V_{n,out,M1}^2} + \overline{V_{n,out,RL}^2}}{A_v^2 \cdot \overline{V_{n,Rs}^2}} =$

1 a) cont.

$$= 1 + \frac{kT\gamma \frac{R_L^2}{R_s} + 4kTR_L}{\left(\frac{R_L}{2R_s}\right)^2 \cdot 4kTR_s} = 1 + \frac{}{kT\frac{R_L^2}{R_s}} =$$

$$= \boxed{1 + \gamma + 4\frac{R_s}{R_L}} \quad NF$$

1 b) For gate noise, add another voltage source on the gate: $\overline{V_{n\,out,M1g}^2} = \frac{4kTR_G}{3} \cdot \left(\frac{R_L}{2R_s}\right)^2 =$

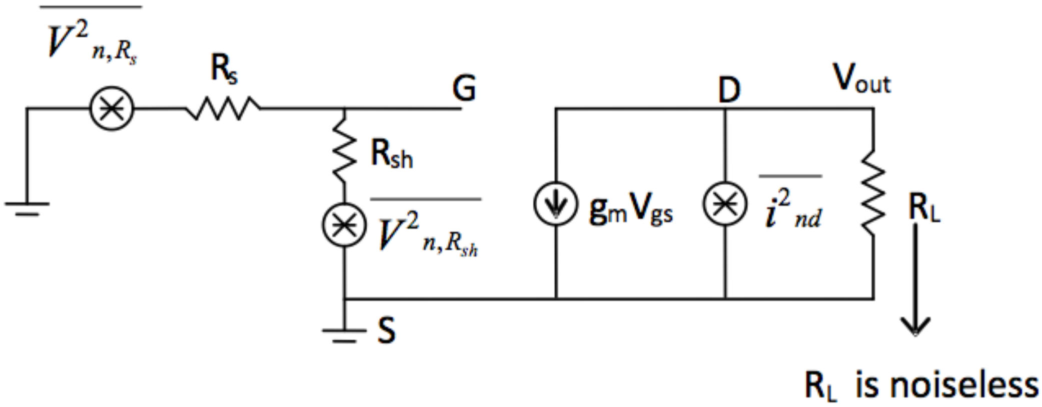
$$= \frac{kT \cdot R_G \cdot R_L^2}{3R_s^2}$$

$$\Rightarrow NF = 1 + \frac{kT\gamma \frac{R_L^2}{R_s} + \frac{kTR_G \cdot R_L^2}{3R_s^2} + 4kTR_L}{kT\frac{R_L^2}{R_s}} =$$

$$\boxed{1 + \gamma + \frac{R_G}{3R_s} + 4\frac{R_s}{R_L}} \quad NF$$

2.

a.



Gain:

$$Gain\big|_{Gate} = -g_m R_L$$

$$A = -g_m R_L \left(\frac{R_{sh}}{R_s + R_{sh}} \right)$$

Zin: When the gate capacitance is not included in the small-signal model (above), the input impedance is purely resistive: $Z_{in} = R_{sh}$.

Noise Figure:

$$F = \frac{Total\ output\ noise\ power}{Output\ noise\ due\ to\ input\ source}$$

$$\overline{V^2_{n,R_s}} = 4kTR_s \qquad \overline{V^2_{n,R_{sh}}} = 4kTR_{sh} \qquad \overline{i^2_{nd}} = 4kT\gamma g_m$$

Using superposition, only one noise source is considered at a time and other sources should be shorted (voltage noise source) / open (current noise source).

$$\overline{V^2_{no,Rs}} = \overline{V^2_{n,Rs}} \times g_m^2 R_L^2 \times \left(\frac{R_{sh}}{R_s + R_{sh}} \right)^2$$

$$\overline{V^2_{no,R_{sh}}} = \overline{V^2_{n,R_{sh}}} \times g_m^2 R_L^2 \times \left(\frac{R_s}{R_s + R_{sh}} \right)^2$$

$$\overline{V^2_{no,d}} = \overline{i^2_{nd}} \times R_L^2$$

$$F = \frac{\overline{V^2_{no,Rs}} + \overline{V^2_{no,R_{sh}}} + \overline{V^2_{no,d}}}{\overline{V^2_{no,R_s}}} = 1 + \frac{\overline{V^2_{no,R_{sh}}} + \overline{V^2_{no,d}}}{\overline{V^2_{no,R_s}}}$$

$$F = 1 + \frac{4kTR_{sh} \dfrac{g_m^2 R_L^2 R_s^2}{(R_s + R_{sh})^2}}{4kTR_s \dfrac{g_m^2 R_L^2 R_{sh}^2}{(R_s + R_{sh})^2}} + \frac{4kT\gamma g_m R_L^2}{4kTR_s \dfrac{g_m^2 R_L^2 R_{sh}^2}{(R_s + R_{sh})^2}} =$$

$$1 + \frac{R_{sh} \dfrac{g_m^2 R_L^2 R_s^2}{(R_s + R_{sh})^2}}{R_s \dfrac{g_m^2 R_L^2 R_{sh}^2}{(R_s + R_{sh})^2}} + \frac{\gamma g_m R_L^2}{R_s \dfrac{g_m^2 R_L^2 R_{sh}^2}{(R_s + R_{sh})^2}} = 1 + \frac{R_{sh} g_m^2 R_L^2 R_s^2}{R_s g_m^2 R_L^2 R_{sh}^2} + \frac{\gamma g_m R_L^2 (R_s + R_{sh})^2}{R_s g_m^2 R_L^2 R_{sh}^2} =$$

$$1 + \frac{R_s}{R_{sh}} + \frac{\gamma (R_s + R_{sh})^2}{g_m R_s R_{sh}^2}$$

Incorrect in first version!

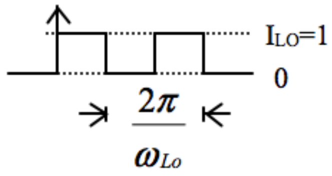
b. Best LNA performance in the system is achieved when Z_{in} = 50 Ω => $R_s = R_{sh}$ = 50 Ω.

$$F = 1 + 1 + \frac{\gamma (R_s + R_{sh})^2}{g_m R_s R_{sh}^2} = 2 + \frac{4\gamma}{g_m R_s}$$

Incorrect in first version!

3.

LO-IF feedthrough: measured level of the 900-MHz output component in the absence of an RF signal.



$$i_{LO}^{+}(t)=\frac{1}{2}+\frac{2}{\pi}\cos\omega_{LO}(t)-\frac{2}{3\pi}\cos 3\omega_{LO}(t)+\frac{2}{5\pi}\cos 5\omega_{LO}(t)-...$$
$$i_{LO}^{-}(t)=\frac{1}{2}-\frac{2}{\pi}\cos\omega_{LO}(t)+\frac{2}{3\pi}\cos 3\omega_{LO}(t)-\frac{2}{5\pi}\cos 5\omega_{LO}(t)+...$$

$$i_{RF}(t)=I_1+I_{RF}\cos\omega_{RF}t$$

No RF signal: $I_{RF}=0$ => $i_{RF}(t)=I_1$

The output current at IF is given by:

$$i_{IF}^{+}(t)=i_{LO}^{+}(t)\times i_{RF}(t)=\left[\frac{1}{2}+\frac{2}{\pi}\cos\omega_{LO}(t)-\frac{2}{3\pi}\cos 3\omega_{LO}(t)+\frac{2}{5\pi}\cos 5\omega_{LO}(t)-...\right]\cdot(I_1)$$
$$=\frac{I_1}{2}+\frac{2I_1}{\pi}\cos\omega_{LO}(t)$$
$$i_{IF}^{-}(t)=i_{LO}^{-}(t)\times i_{RF}(t)=\left[\frac{1}{2}-\frac{2}{\pi}\cos\omega_{LO}(t)+\frac{2}{3\pi}\cos 3\omega_{LO}(t)-\frac{2}{5\pi}\cos 5\omega_{LO}(t)+...\right]\cdot(I_1)$$
$$=\frac{I_1}{2}-\frac{2I_1}{\pi}\cos\omega_{LO}(t)$$

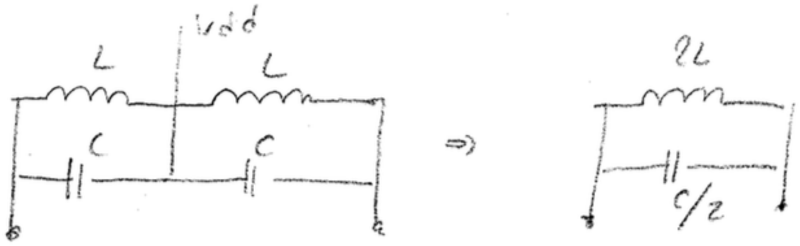
$$i_{IF}(t)=i_{IF}^{+}(t)-i_{IF}^{-}(t)=\frac{4I_1}{\pi}\cos\omega_{LO}(t)$$

$$v_{IF}(t)=i_{IF}(t)\times R_p=\frac{4}{\pi}I_1R_p\cdot\cos\omega_{LO}(t)$$
where R_p is the parallel resistance, which models the inductor loss.

$R_p=Q\omega_o L_p$ => LO-IF feedthrough $=\frac{4}{\pi}I_1R_p=\frac{4}{\pi}I_1Q\omega_{LO}L_p$

4.

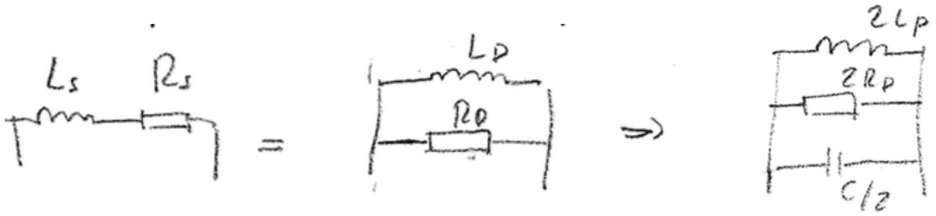
Equivalent circuits of the tank with no resistive loss:



a. Oscillation frequency with losses: $\omega_o = \frac{1}{\sqrt{2L * C/2}}$

=> f ≈ 2.055 GHz.

b. To include losses and calculate a more accurate oscillation frequency, include parallel resistor Rp and use corrected value Lp:



$$\omega_o = \frac{1}{\sqrt{2L_p * C/2}}$$

$L_p = (1 + 1/Q^2) * L_s$

$$\omega_o = \frac{Q}{\sqrt{Q^2 + 1}} \times \frac{1}{\sqrt{L_S C}}$$

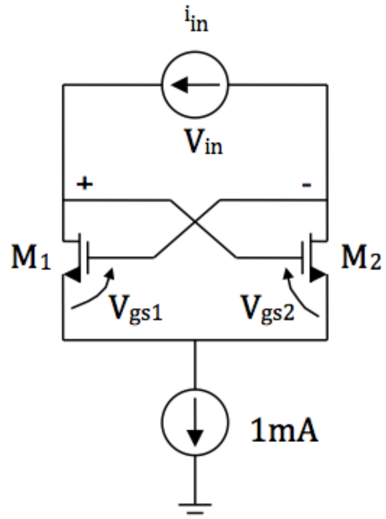
=> f ≈ 1.993 GHz.

The frequency is about 62 MHz, or 3 %, lower.

c. First, we need to determine the required R_p for oscillations:

The parallel resistance $R_P = Q * \omega_0 * L_p$ = 4 * (2*pi*1993E6) * (1+1/16)*2E-9 = 106.4 Ω.
To ensure oscillations, M1/M2 circuit needs to provide an $R_{negative}$ of -106.4 Ω.

The negative resistance seen by RLC:



$$i_{in} = g_{m1}V_{gs1}, \quad -i_{in} = g_{m2}V_{gs2}$$

$$V_{in} = V_{gs2} - V_{gs1} = \frac{-i_{in}}{g_{m2}} - \frac{i_{in}}{g_{m1}} = -i_{in}\left(\frac{1}{g_{m2}} + \frac{1}{g_{m1}}\right)$$

$$R_{in} = \frac{V_{in}}{i_{in}} = -\left(\frac{1}{g_{m2}} + \frac{1}{g_{m1}}\right) = -\frac{2}{g_m} = -R_P$$

Since $g_m = g_{m1} = g_{m2} \approx \sqrt{2\mu_n C_{ox} I_D (W/L)}$

$$\mu_n C_{ox} = 200\,\mu A/V^2, \; I_D = \frac{1\,mA}{2} = 0.5\,mA$$

then $\frac{2}{R_P} = \sqrt{2\mu_n C_{ox} I_D (W/L)}$

$$(W/L) = \frac{2}{\mu_n C_{ox} I_D R_P^2}$$

$$(W/L) \cong 1765$$

With L=0.06 um => W = 106 um.

5.

a. closed-loop transfer function

$$F(s) = \frac{\frac{1}{sC_1}}{\frac{1}{sC_1} + R_1} = \frac{1}{sR_1C_1 + 1}$$

The close loop Transfer Function:

$$\varphi_{out} = \frac{K_{VCO}K_{PD}}{s}\left(\varphi_{in} - \frac{\varphi_{out}}{M}\right)\left(\frac{1}{sR_1C_1 + 1}\right)$$

$$\frac{\varphi_{out}}{\varphi_{in}}(s) = \frac{K_{VCO}K_{PD}M}{s(sR_1C_1 + 1)M + K_{VCO}K_{PD}}$$

$$= \frac{K_{VCO}K_{PD}M}{s^2R_1C_1M + sM + K_{VCO}K_{PD}} = \frac{\frac{K_{VCO}K_{PD}}{R_1C_1}}{s^2 + s\frac{1}{R_1C_1} + \frac{K_{VCO}K_{PD}}{R_1C_1M}} = \frac{\frac{K_{VCO}K_{PD}}{R_1C_1}}{s^2 + 2\xi\omega_n s + \omega_n^{\,2}}$$

b. damping factor ζ

$$\xi = \frac{1}{2}\sqrt{\frac{\omega_{LPF}M}{K_{VCO}K_{PD}}}$$

c. natural frequency ω_n

$$\omega_n = \sqrt{\frac{K_{VCO}K_{PD}\omega_{LPF}}{M}}$$

d. loop bandwidth = $\zeta * \omega_n = 1/2 * \omega_{LPF} = 1 / (2RC)$.

6.

a. C B D A E
(0.5 p for each correct amplifier class.)

b. A class-AB power amplifier is biased between class-A and class-B.
$V_{g,bias}$ for a class-A amplifier is selected to some value $>V_{TH}$ to achieve a standby current of $I_{D,max}/2$ (with no RF input signal).
$V_{g,bias}$ for a class-B amplifier is selected to V_{TH} to conduct 50 % of the cycle (conduction angle of 180°) (assuming a symmetrical input swing signal +/- Vp). The standby current is 0 (with no RF input signal).
$V_{g,bias}$ for a class-AB is selected somewhere between these two values, which can also be expressed as a conduction angle between ∏ and 2*∏.

c. Efficiency vs. linearity.
("Trade-off" means that we have two properties that goes better/worse, worse/better. Need both properties.)

Missed in first version!

d. To determine the optimum load for best PA performance (which is often maximum linear output power (P-1dB), or maximum efficiency). When doing it for the input (which is rather linear from the beginning), this separate measurement may be called "source-pull".