

# ANSWERS

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## TSEK03

### RADIO FREQUENCY INTEGRATED CIRCUITS

Date: 2014-03-20  
Time: 8-12  
Location: U1  
Aids: Calculator, Dictionary  
Teachers: Behzad Mesgarzadeh (5719)  
Amin Ojani (2815)

12 points are required to pass.

12-16 : 3

16-20 : 4

20-24 : 5

**Please start each new problem at the top of a page!**  
**Only use one side of each paper!**

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1)

(a)

$$4kTR = 33 \times 10^{-17} \Rightarrow R \approx 20 \text{ K}\Omega$$

The transfer function of this filter is

$$H(s) = \frac{1}{1 + RCs}$$

The peak value of the transfer function is 1, then:

$$\Delta f = \frac{1}{|H_{pk}|^2} \int_0^\infty |H(f)|^2 df = \frac{1}{1} \int_0^\infty \frac{1}{1 + 4\pi^2 f^2 R^2 C^2} df \Rightarrow$$
$$\Delta f = \frac{1}{2\pi RC} \tan^{-1}(2\pi RCf) \Big|_0^\infty = \frac{1}{4RC} = 50 \times 10^6 \xrightarrow{R=20 \text{ K}\Omega} C = 0.25 \text{ pF}$$

2)

(a)

By applying a test voltage source to input node, we can determine the input impedance as

$$Z_{in} = sL_1 + \frac{1}{sC_{GS1}} + \frac{g_{m1}}{C_{GS1}} L_1 = j\omega L_1 + \frac{1}{j\omega C_{GS1}} + \frac{g_{m1}}{C_{GS1}} L_1$$

Putting the imaginary part equal to zero results in:

$$\omega = \frac{1}{\sqrt{L_1 C_{GS1}}} = 25 \times 10^9 \rightarrow f \approx 4 \text{ GHz}$$

(b)

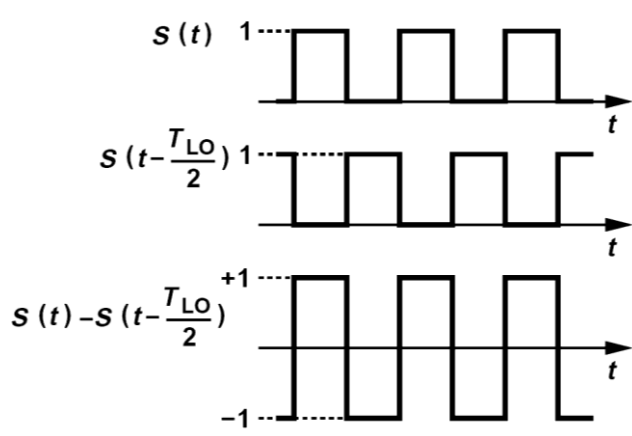
For matching:

$$\frac{g_{m1}}{C_{GS1}} L_1 = 50 \rightarrow g_{m1} = 31.25 \text{ mA/V}$$

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3)

(a)



$$V_{out}(t) = I_{RF}R[S(t) - S(t - T_{LO}/2)] = I_{RF}R \cdot \frac{4}{\pi} \cos(\omega_{LO}t) + \dots$$

If  $V_{RF} = A_{RF} \cos(\omega_{RF}t)$ , then by ignoring the higher order terms:

$$V_{out}(t) = \frac{4}{\pi} g_{m3} R A_{RF} \cos(\omega_{RF}t) \cos(\omega_{LO}t)$$

$$\rightarrow V_{IF} = \frac{2}{\pi} g_{m3} R A_{RF} \cos((\omega_{RF} - \omega_{LO})t)$$

Therefore the conversion gain is:

$$G_C = \frac{V_{IF}}{A_{RF}} = \frac{2}{\pi} g_{m3} R$$

(b)

For a sinusoidal LO signal, drain currents of  $M_1$  and  $M_2$  will remain approximately equal for a period of  $\Delta T$  in each half cycle, appearing as common mode output which is canceled differentially. Since it happens twice in each period then the new conversion gain is:

$$G_C = \frac{V_{IF}}{A_{RF}} = \frac{2}{\pi} g_{m3} R \left(1 - \frac{2\Delta T}{T}\right)$$

$$\Delta T = 0.1T \rightarrow G_{C,sin} = 0.8G_{C,square} \rightarrow G_{C,sin}(dB) = G_{C,square}(dB) - 1.93$$

The conversion gain reduces by 1.93 dB.

4)

(a)

No, it is not stable. The open-loop transfer function has two poles at origin. Each of these poles contributes  $90^\circ$  to the total phase shift creating totally  $180^\circ$  frequency-dependent phase shift for all frequencies. According to Barkhausen criteria, at gain crossover point, if the total phase shift around the loop is  $360^\circ$  then the feedback system will oscillates.

(b)

Closed-loop transfer function is

$$\frac{Y}{X}(s) = \frac{K^2}{s^2 + K^2}$$

Then in time domain:

$$\frac{d^2y}{dt^2} + K^2y = K^2x$$

For  $x = 0 \rightarrow y = A \cos(Kt + \phi)$

It is in line with our answer in party (a) since it shows that the feedback system oscillates generating a sinusoidal output with frequency of  $K$ , at which the loop gain ( $K^2/s^2$ ) equals to 1 (gain crossover frequency).

5)

(a) Type I:

$$H(s) = \frac{\phi_{out}}{\phi_{in}}(s) = \frac{K_{PD}K_{VCO}}{R_1C_1s^2 + s + K_{PD}K_{VCO}}$$

(b) For slow variations ( $s \approx 0$ ) and  $H(s) = 1$ . Then output phase tracks the input.

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