## EXAMINATION IN

## TSEK03

## Radio Frequency Integrated Circuits

Date:
2014-03-20
Time:
8-12
Location: U1
Aids: Calculator, Dictionary
Teachers: Behzad Mesgarzadeh (5719)
Amin Ojani (2815)
12 points are required to pass.
12-16:3
16-20 : 4
20-24:5
Please start each new problem at the top of a page! Only use one side of each paper!

1) The mean square thermal noise density of a resistor in the room temperature is $33 \times 10^{-17} V^{2} / H z$. If this resistor is used in a first-order $R C$ filter as shown in Fig. 1, and the noise bandwidth of the $R C$ filter is 50 MHz , calculate the value of $C$ in Fig.1. Present the details of your calculations.


Fig. 1. A single-pole RC filter.

## Hints:

i) Boltzmann's constant is $1.38 \times 10^{-23} \mathrm{~J} / \mathrm{K}$
ii) $\int \frac{d x}{1+x^{2}}=\tan ^{-1} x$
iii) Noise bandwidth: $\Delta f=\frac{1}{\left|H_{p k}\right|^{2}} \int_{0}^{\infty}|H(f)|^{2} d f$
2) Input stage of a single-ended LNA is shown in Fig. 2. Assume that $C_{G S 1}=1 \mathrm{pF}$, $L_{1}=1.6 \mathrm{nH}$, and $\lambda=\gamma=0$.
(a) Calculate the frequency at which the input impedance is purely resistive (a real value).
(b) Calculate the transconductance of the transistor $\left(g_{m 1}\right)$, to match with a $50-\Omega$ source resistance.


Fig. 2. Input stage of a single-ended LNA.
3) A single-balanced mixer is shown in Fig. 3. Ignore channel length modulation.


Fig. 3. A single-balanced mixer.
(a) If LO signal is a square wave toggling between 0 and 1 with $50 \%$ duty cycle and LO switching is abrupt, derive an expression for the conversion gain of this mixer.
(b) If LO signal is a sine wave varying between 0 and 1 , drain currents of $M_{1}$ and $M_{2}$ will remain approximately equal for a period of $\Delta \mathrm{T}$ in each half cycle reducing the conversion gain. If $\Delta \mathrm{T}$ is $10 \%$ of the LO signal period, calculate the conversion gain reduction in dB .

## Hint:

For a square wave LO signal toggling between -1 and 1 :
$V_{L O}(t)=\frac{4}{\pi} \cos \omega_{L O}(t)-\frac{4}{3 \pi} \cos 3 \omega_{L O}(t)+\frac{4}{5 \pi} \cos 5 \omega_{L O}(t)-\ldots$
4) Figure 4 shows a unity gain feedback system with two identical ideal integrators. The transfer function of each integrator in Laplace domain is $H(s)=K / s$.
(a) Is this feedback system stable? Motivate your answer using Barkhausen criteria.
(b) Write a differential equation describing this feedback system and solve it for $X=0$. Is this result consistent with your answer in part (a)? Why?


Fig. 4. Two ideal integrators inside a unity gain feedback.
5) Figure 5 shows a block level description of a PLL.
(a) Determine the closed-loop transfer function (i.e., $\left.\frac{\Phi_{\text {out }}}{\Phi_{\text {in }}}(s)\right)$ and the type of the PLL.
(b) Prove that for slow input phase variations the output tracks the input.


Fig. 5. Block diagram of a PLL.

## TRANSISTOR EQUATIONS



PMOS


## NMOS

- Cutoff:

$$
\mathrm{I}_{\mathrm{D}}=0 \quad\left(\mathrm{~V}_{\mathrm{GS}}<\mathrm{V}_{\mathrm{TN}}\right)
$$

- Linear mode:
$I_{D}=\mu_{n} C_{o x} \frac{W}{L}\left(\left(V_{G S}-V_{T N}\right) V_{D S}-\frac{V_{D S}^{2}}{2}\right) \quad\left(\mathrm{V}_{\mathrm{GS}}>\mathrm{V}_{\mathrm{TN}}\right)$ and $\left(\mathrm{V}_{\mathrm{DS}}<\mathrm{V}_{\mathrm{GS}}-\mathrm{V}_{\mathrm{TN}}\right)$
- Saturation mode:
$I_{D}=\frac{1}{2} \mu_{n} C_{o x} \frac{W}{L}\left(V_{G S}-V_{T N}\right)^{2}\left(1+\lambda V_{D S}\right) \quad\left(\mathrm{V}_{\mathrm{GS}}>\mathrm{V}_{\mathrm{TN}}\right)$ and $\left(\mathrm{V}_{\mathrm{DS}}>\mathrm{V}_{\mathrm{GS}}-\mathrm{V}_{\mathrm{TN}}\right)$


## PMOS

- Cutoff: $\quad \mathrm{I}_{\mathrm{D}}=0 \quad\left(\mathrm{~V}_{\mathrm{GS}}<\left|\mathrm{V}_{\mathrm{TP}}\right|\right)$
- Linear mode:

$$
I_{D}=\mu_{p} C_{o x} \frac{W}{L}\left(\left(V_{S G}-\left|V_{T P}\right|\right) V_{S D}-\frac{V_{S D}^{2}}{2}\right) \quad\left(\mathrm{V}_{\mathrm{GS}}>\left|\mathrm{V}_{\mathrm{TP}}\right|\right) \text { and }\left(\mathrm{V}_{\mathrm{SD}}<\mathrm{V}_{\mathrm{SG}}-\left|\mathrm{V}_{\mathrm{TP}}\right|\right)
$$

- Saturation mode:

$$
I_{D}=\frac{1}{2} \mu_{p} C_{o x} \frac{W}{L}\left(V_{S G}-\left|V_{T P}\right|\right)^{2}\left(1+\lambda V_{S D}\right) \quad\left(\mathrm{V}_{\mathrm{GS}}>\left|\mathrm{V}_{\mathrm{TP}}\right|\right) \text { and }\left(\mathrm{V}_{\mathrm{SD}}>\mathrm{V}_{\mathrm{SG}}-\left|\mathrm{V}_{\mathrm{TP}}\right|\right)
$$

