LINKÖPING UNIVERSITY
Behzad Mesgarzadeh
Amin Ojani
Electronic Devices, Department of Electrical Engineering

ANSWERS
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TSEK03
RADIO FREQUENCY INTEGRATED CIRCUITS

Date:	2013-06-05
Time:	14-18
Location:	TER3
Aids:	Calculator, Dictionary
Teachers:	Behzad Mesgarzadeh (5719)
	Amin Ojani (2716)

12 points are required to pass.

12-16 : 3

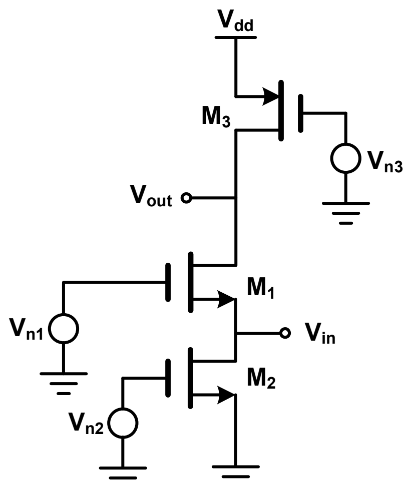
16-20 : 4

20-24 : 5

Please start each new problem at the top of a page!
Only use one side of each paper!

1)

There are three flicker noise sources as shown below.



To determine the output noise we short the input to ground (M_2 has no effect). Then:

$$\overline{V_{n,out}^2} = \left(\overline{V_{n1}^2} g_{m1}^2 + \overline{V_{n3}^2} g_{m3}^2\right) \cdot (r_{o1} \parallel r_{o3})^2$$

Since $g_m \gg 1/r_o$ and $\gamma = 0$, the gain of this circuit is:

$$A = g_{m1}(r_{o1} \parallel r_{o3})$$

Then we can determine the input referred noise as:

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n1}^2} g_{m1}^2 + \overline{V_{n3}^2} g_{m3}^2}{g_{m1}^2}$$

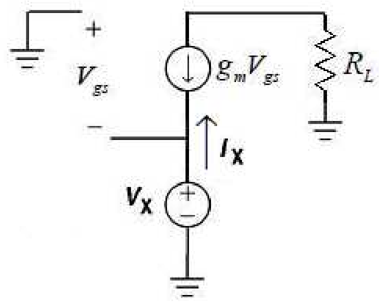
Repacing the values of $\overline{V_{n1}^2}$ and $\overline{V_{n3}^2}$ gives:

$$\overline{V_{n,in}^2} = \frac{1}{fC_{ox}}\left[\frac{K_n}{(WL)_2} + \frac{g_{m3}^2 K_p}{(WL)_3}\right]$$

2)

(a)

__**Input Impedance:**__



$$I_x + g_m V_{gs} = 0$$
$$V_x = V_s$$
$$V_{gs} = V_g - V_s = -V_s = -V_x$$
$$I_x - g_m V_x = 0$$
$$Z_{in} = \frac{V_x}{I_x} = \frac{1}{g_m}$$

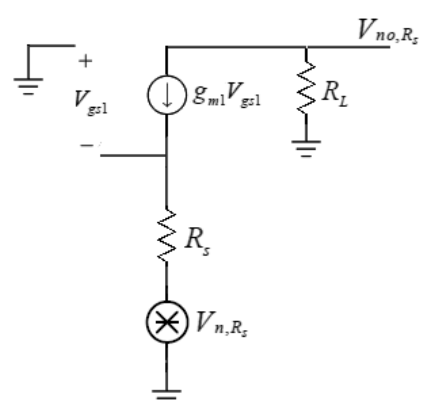
(b)

Noise Factor:

There are totally three noisy components to be considered which are the source resistance R_s and NMOS transistors M1 and M2. So the noise factor of the LNA is:

$$\text{Noise Factor} = \frac{\overline{V^2_{no,R_s}} + \overline{V^2_{no,M_1}} + \overline{V^2_{no,M_2}}}{\overline{V^2_{no,R_s}}}$$

Calculation of $\overline{V^2_{no,R_s}}$:



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$$V_{gs}=V_g-V_s=-V_s$$

$$\frac{V_{n,R_s}-V_s}{R_s}=-g_mV_{gs}=g_mV_s$$

$$V_s=V_{n,R_s}\left(\frac{1}{g_mR_s+1}\right)\text{.............................(1)}$$

$$g_mV_{gs}=-g_mV_s=\frac{-V_{no,R_s}}{R_L}\text{....................(2)}$$

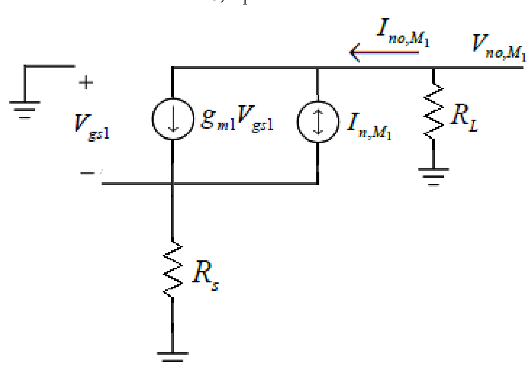
Apply (1) into (2) gives:

$$V_{no,R_s}=\frac{g_mR_L}{1+g_mR_s}V_{n,R_s}$$

$$\overline{V^2{}_{n,R_s}}=4KTR_s$$

$$\overline{V^2{}_{no,R_s}}=\left(\frac{g_{m_1}R_L}{1+g_{m_1}R_s}\right)^2\times\overline{V^2{}_{n,R_s}}=\left(\frac{g_{m_1}R_L}{1+g_{m_1}R_s}\right)^2 4KTR_s$$

Calculation of $\overline{V^2{}_{no,M_1}}$:



$$g_{d0}\approx g_m$$

$$\overline{I^2{}_{n,M_1}}=4KT\gamma g_{m_1}$$

$$V_{no,M_1}=-I_{out}R_L\text{..(1)}$$

$$I_{no,M_1}=g_{m_1}V_{gs}+I_{n,M_1}=-g_{m_1}V_s+I_{n,M_1}\text{...............(2)}$$

$$\frac{V_s}{R_s}=\frac{-V_{no,M_1}}{R_L}\text{..(3)}$$

Applying (3) into (2) gives:

$$I_{no,M_1}=-g_{m_1}\frac{-R_s}{R_L}V_{no,M_1}+I_{n,M_1}\text{........................(4)}$$

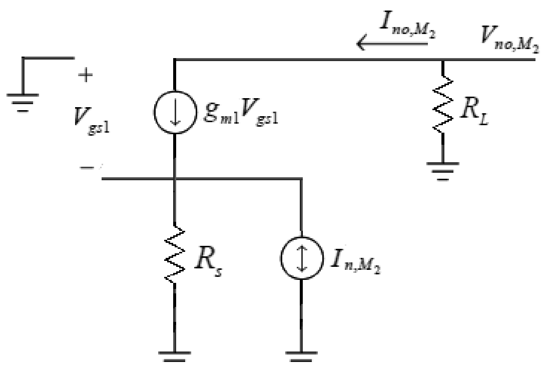
Applying (4) into (1) gives:

$$V_{no,M_1} = \left(g_{m_1} \frac{R_s}{R_L} V_{no,M_1} + I_{n,M_1} \right) R_L$$

$$V_{no,M_1} = \left(\frac{-R_L}{1+g_{m_1} R_s} \right) I_{n,M_1}$$

$$\overline{V^2_{no,M_1}} = \left(\frac{-R_L}{1+g_{m_1} R_s} \right)^2 \overline{I^2_{n,M_1}} = \left(\frac{-R_L}{1+g_{m_1} R_s} \right)^2 4KT\gamma g_{m_1}$$

Calculation of $\overline{V^2_{no,M_2}}$:



$$g_{d0} \approx g_m$$

$$\overline{I^2_{n,M_2}} = 4KT\gamma g_{m_2}$$

$$V_{no,M_2} = -I_{no,M_2} R_L \text{..(5)}$$

$$I_{no,M_2} = \frac{V_s}{R_s} + I_{n,M_2} \text{..(6)}$$

$$\frac{-V_{no,M_2}}{R_L} = g_{m_1} V_{gs} = -g_{m_1} V_s$$

$$V_s = V_{no,M_2} \frac{1}{g_{m_1} R_L} \text{.......................................(7)}$$

Applying (7) into (6) gives:

$$I_{no,M_2} = V_{no,M_2} \frac{1}{g_{m_1} R_L R_s} + I_{n,M_2} \text{..........................(8)}$$

Applying (8) into (5) gives:

$$V_{n,M_2} = -R_L\left(V_{n,M_2}\frac{1}{g_{m_1}R_LR_s} + I_{n,M_2}\right)$$

$$V_{n,M_2} = \left(\frac{-g_{m_1}R_LR_s}{g_{m_1}R_s+1}\right)I_{n,M_2}$$

$$\overline{V^2{}_{n,M_2}} = \left(\frac{-g_{m_1}R_LR_s}{g_{m_1}R_s+1}\right)^2\overline{I^2{}_{n,M_2}} = \left(\frac{-g_{m_1}R_LR_s}{g_{m_1}R_s+1}\right)^2 4KT\gamma g_{m_2}$$

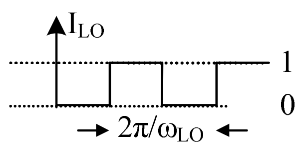
Total Noise Factor of the LNA:

$$F = \frac{\overline{V^2{}_{no,R_s}} + \overline{V^2{}_{no,M_1}} + \overline{V^2{}_{no,M_2}}}{\overline{V^2{}_{no,R_s}}} = 1 + \frac{\overline{V^2{}_{no,M_1}}}{\overline{V^2{}_{no,R_s}}} + \frac{\overline{V^2{}_{no,M_2}}}{\overline{V^2{}_{no,R_s}}} =$$

$$1 + \frac{\left(\frac{-R_L}{1+g_{m_1}R_s}\right)^2 4KT\gamma g_{m_1}}{\left(\frac{g_{m_1}R_L}{1+g_{m_1}R_s}\right)^2 4KTR_s} + \frac{\left(\frac{-g_{m_1}R_LR_s}{1+g_{m_1}R_s}\right)^2 4KT\gamma g_{m_2}}{\left(\frac{g_{m_1}R_L}{1+g_{m_1}R_s}\right)^2 4KTR_s} = 1 + \frac{\gamma}{g_{m_1}R_s} + \gamma g_{m_2}R_s$$

3)

a)



$$i_{LO}(t) = \frac{1}{2} + \frac{2}{\pi}\cos\omega_{LO}(t) - \frac{2}{3\pi}\cos 3\omega_{LO}(t) + \frac{2}{5\pi}\cos 5\omega_{LO}(t) - \ldots$$

$$i_{RF}(t) = I_{bias} + I_{RF}\cos\omega_{RF}(t)$$

$$i_{IF}(t) = i_{LO}(t)\times i_{RF}(t) = \left(\frac{1}{2} + \frac{2}{\pi}\cos\omega_{LO}(t) - \ldots\right)\times\left(I_{bias} + I_{RF}\cos\omega_{RF}(t)\right)$$

$$= \underbrace{\frac{I_{bias}}{2}}_{DC} + \underbrace{\frac{2I_{bias}}{\pi}\cos\omega_{LO}(t)}_{LO\ \ feedthrough} + \underbrace{\frac{I_{RF}}{2}\cos\omega_{RF}(t)}_{RF\ \ feedthrough} + \frac{I_{RF}}{\pi}\left[\underbrace{\cos(\omega_{RF}-\omega_{LO})(t)}_{IF} + \underbrace{\cos(\omega_{RF}+\omega_{LO})(t)}_{HF\ (to\ be\ filtered)}\right]$$

b)

$$v_{IF}(t) = R_D \cdot \frac{I_{RF}}{\pi}\cos(\omega_{RF} - \omega_{LO})t$$

$$v_{RF}(t) = \frac{i_{RF}(t)}{\frac{g_{m2}}{1+g_{m2}R_{on1}}} = \frac{I_{RF}\cos\omega_{RF}t}{\frac{g_{m2}}{1+g_{m2}R_{on1}}} \quad \Rightarrow$$

$$G_c = \left|\frac{v_{IF}(t)}{v_{RF}(t)}\right| = \left|\frac{R_D \cdot \frac{I_{RF}}{\pi}\cos(\omega_{RF} - \omega_{LO})t}{\frac{I_{RF}\cos\omega_{RF}t}{\frac{g_{m2}}{1+g_{m2}R_{on1}}}}\right| = \frac{1}{\pi}\frac{g_{m2}}{1+g_{m2}R_{on1}}R_D$$

c)

M_1 is an ideal switch with on resistance equal to zero => no noise contribution

Noise sources are M_2, R_S, and R_D:

$$\overline{V^2_{n,out\ \ M2}} = R_D^{\ 2} \times \overline{i^2_{n,out\ \ M2}} = 4KT\gamma g_{m2}R_D^{\ 2}$$

$$\overline{V^2_{n,out\ \ R_D}} = 4KTR_D$$

$$\overline{V^2_{n,out\ \ R_S}} = 4KTR_S(g_{m2}R_D)^2$$

Since a perfect switch driven by an ideal square wave generates odd harmonics, the gain for noise and signal is different.

$$Y(\omega) = \sum_k \frac{2}{k\pi}X(\omega + k\omega_0)\,.$$

Gain for noise added by switching: $Y(\omega) = \sum_k |a_k|^2 = 1$

Gain for signal added by switching (only fundamental): $|a_1|^2 = \frac{4}{\pi^2}$

To calculate the noise figure:

$$F = \frac{SNR_{in}}{SNR_{out}} = \frac{S_{in}/N_{in}}{S_{out}/N_{out}} = \frac{S_{in}}{S_{out}}\cdot\frac{N_{out}}{N_{in}} = \frac{1}{G_c}\cdot\frac{N_{out}}{N_{in}}$$

$$F = \frac{1}{G_c^{\ 2}} \cdot \frac{\overline{V^2_{n,out\ R_S}} + \overline{V^2_{n,out\ M2}} + \overline{V^2_{n,out\ R_D}}}{\overline{V^2_{n,\ R_S}}}$$

$$= \frac{\pi^2}{(g_{m2}R_D)^2} \cdot \frac{4KTR_S(g_{m2}R_D)^2 + 4KTR_D + 4KT\gamma g_{m2}R_D^{\ 2}}{4KTR_S}$$

$$= \pi^2\left(1 + \frac{1}{(g_{m2})^2 R_D R_S} + \frac{\gamma}{g_{m2}R_S}\right)$$

4)

(a) $R_S = \frac{\omega_0 L_S}{Q} = \frac{2\pi(2.4\times10^9)(5\times10^{-9})}{10} = 7.54\,\Omega$

$R_p = (1+Q^2)2R_S = 1.523\,\mathrm{k}\Omega$

$R_{neg} = -\frac{2}{g_m}$

For oscillation $R_p = \left|R_{neg}\right|$

$1.523\times10^3 = \frac{2}{g_m}$ and, $g_m = 1.313$ mA/V

$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D}$ and, $I_D = \frac{I_{bias}}{2}$, therefore

$\frac{W}{L} = \frac{g_m^2}{\mu_n C_{ox} I_{bias}} = 7.84$

(b)

The parallel inductance is calculated as $L_P = 2L_S\left(1+\frac{1}{Q^2}\right)$. However, since the Q is relatively large in this example, L_p can be approximated by $L_P \approx 2L_S$. Therefore,

$f_0 \approx \frac{1}{2\pi\sqrt{2L_S C}}$

For oscillation frequency of 2.4 GHz, $C = \frac{1}{4\pi^2(2L_S)f_0^2}$ = 0.440 pF

while for oscillation frequency of 2.5 GHz, $C = \frac{1}{4\pi^2(2L_S)f_0^2}$ = 0.405 pF

(c)

$$L_{eq} = \frac{2L_{add}L_S}{L_{add} + 2L_S} = \frac{(5\times10^{-9})(10\times10^{-9})}{5\times10^{-9} + 10\times10^{-9}} = 3.33 \text{ nH}$$

$$f_0 = \frac{1}{2\pi\sqrt{L_{eq}C}} = \frac{1}{2\pi\sqrt{(3.33\times10^{-9})0.44\times10^{-12}}} = 4.16 \text{ GHz}$$

$$\% \text{ change} = \frac{4.16 - 2.4}{2.4}\times100 = 73.3 \text{ \% increase}$$

5)

(a) Type I:

$$H(s) = \frac{\phi_{out}}{\phi_{in}}(s) = \frac{K_{PD}K_{VCO}}{R_1C_1s^2 + s + K_{PD}K_{VCO}}.$$

(b) For slow variations ($s \approx 0$) and $H(s) = 1$. Then output phase tracks the input.