## ExAMINATION IN

## Radio Frequency Integrated Circuits

Date:<br>2013-06-05<br>Time:<br>14-18<br>Location: TER3<br>Aids: Calculator, Dictionary<br>Teachers: Behzad Mesgarzadeh (5719)<br>Amin Ojani (2716)

12 points are required to pass.
12-16: 3
16-20 : 4
20-24 : 5
Please start each new problem at the top of a page! Only use one side of each paper!

1) Figure 1 shows an amplifier schematic. Determine the input-referred $1 / f$ noise voltage. Ignore all the thermal noise sources. Assume $g_{m} \gg 1 / r_{o}, \gamma=0$, and $\lambda \neq 0$.

Hint: $\overline{V_{n}^{2}}=\frac{K}{C_{o x} W L} \cdot \frac{1}{f}$


Fig. 1. An amplifier schematic.
2) Consider the wideband common-gate low noise amplifier (LNA) shown in Fig. 2. $R_{S}$ is the input source resistance. Assume that the transistors are long-channel devices with $\lambda_{n}=0$. Also assume that $\gamma_{\text {body effect }}=0$.


Fig. 2. A common-gate LNA.
(a) Calculate the input impedance of the LNA. Assume that we can neglect all parasitics associated with the transistors.
(b) Derive an expression for the noise figure of the LNA. Only consider the thermal noise sources and ignore the gate noise of the transistors. Also assume that $R_{L}$ is a noiseless resistor.

Hint: $\overline{i^{2}{ }_{n, M}}=4 K T \gamma g_{m}$
3) The circuit shown in Fig. 3 is a dual-gate mixer used in traditional microwave design. Assume abrupt edges and a $50 \%$ duty cycle for the LO, and neglect channel-length modulation and body effect.


Fig. 3. A dual-gate mixer.
(a) Assume that $M_{1}$ is an ideal switch. Determine all the frequency components which appear at the mixer IF port.
(b) Assume when $M_{1}$ is on, it has an on-resistance of $R_{\text {on1 }}$. Compute the voltage conversion gain of the circuit. Assume $M_{2}$ does not enter the triode region and denote its transconductance by $g_{m 2}$.
(c) Assume that $M_{1}$ is an ideal switch (noise contribution is zero). Derive the expression for the noise figure of the mixer.

## Hints:


i) $i_{L O}(t)=\frac{1}{2}+\frac{2}{\pi} \cos \omega_{L O}(t)-\frac{2}{3 \pi} \cos 3 \omega_{L O}(t)+\frac{2}{5 \pi} \cos 5 \omega_{L O}(t)-\ldots$
ii) $i_{R F}(t)=I_{\text {bias }}+I_{R F} \cos \omega_{R F}(t)$
iii) $\overline{i^{2}{ }_{n, M}}=4 K T \gamma g_{m}$
4) A negative-resistance oscillator operating at 2.4 GHz frequency is shown in Fig. 4. The resonant circuit is implemented using inductor $L=5 \mathrm{nH}$ with $Q=10$ and a variable capacitor $C$. Assume that we can neglect all parasitics associated with the transistors.
(a) What is the minimum width of two identical transistors $M_{1}$ and $M_{2}$ to ensure the oscillation?
(b) How much should the variable capacitance $C$ be varied to enable tuning from 2.4 GHz to 2.5 GHz ?
(c) For a capacitance corresponding to 2.4 GHz oscillation frequency, if an additional inductor of 5 nH is connected in parallel with the capacitor $C$, how much will the oscillation frequency change (in percentage)?


Fig. 4. A negative-resistance oscillator.
5) Figure 5 shows a block level description of a PLL.
(a) Determine the closed-loop transfer function (i.e., $\left.\frac{\Phi_{\text {out }}}{\Phi_{\text {in }}}(s)\right)$ and the type of the PLL.
(b) Prove that for slow input phase variations the output tracks the input.


Fig. 5. Block level description of a PLL.

## TRANSISTOR EQUATIONS

NMOS


PMOS


NMOS

- Cutoff: $\quad \mathrm{I}_{\mathrm{D}}=0 \quad\left(\mathrm{~V}_{\mathrm{GS}}<\mathrm{V}_{\mathrm{TN}}\right)$
- Linear mode:

$$
I_{D}=\mu_{n} C_{o x} \frac{W}{L}\left(\left(V_{G S}-V_{T N}\right) V_{D S}-\frac{V_{D S}^{2}}{2}\right) \quad\left(\mathrm{V}_{\mathrm{GS}}>\mathrm{V}_{\mathrm{TN}}\right) \text { and }\left(\mathrm{V}_{\mathrm{DS}}<\mathrm{V}_{\mathrm{GS}}-\mathrm{V}_{\mathrm{TN}}\right)
$$

- Saturation mode:

$$
I_{D}=\frac{1}{2} \mu_{n} C_{o x} \frac{W}{L}\left(V_{G S}-V_{T N}\right)^{2}\left(1+\lambda V_{D S}\right) \quad\left(\mathrm{V}_{\mathrm{GS}}>\mathrm{V}_{\mathrm{TN}}\right) \text { and }\left(\mathrm{V}_{\mathrm{DS}}>\mathrm{V}_{\mathrm{GS}}-\mathrm{V}_{\mathrm{TN}}\right)
$$

- On-resistance in triode region: $R_{o n}=\frac{1}{\mu_{n} C_{o x} \frac{W}{L}\left(V_{G S}-V_{T N}\right)}$


## PMOS

$$
\text { - Cutoff: } \quad \mathrm{I}_{\mathrm{D}}=0 \quad\left(\mathrm{~V}_{\mathrm{GS}}<\left|\mathrm{V}_{\mathrm{TP}}\right|\right)
$$

- Linear mode:

$$
I_{D}=\mu_{p} C_{o x} \frac{W}{L}\left(\left(V_{S G}-\left|V_{T P}\right|\right) V_{S D}-\frac{V_{S D}^{2}}{2}\right) \quad\left(\mathrm{V}_{\mathrm{GS}}>\left|\mathrm{V}_{\mathrm{TP}}\right|\right) \text { and }\left(\mathrm{V}_{\mathrm{SD}}<\mathrm{V}_{\mathrm{SG}}-\left|\mathrm{V}_{\mathrm{TP}}\right|\right)
$$

- Saturation mode:

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