## ExAMINATION IN

## Radio Frequency Integrated Circuits

Date: 2013-03-14<br>Time:<br>14-18<br>Location: TER1<br>Aids: Calculator, Dictionary<br>Teachers: Behzad Mesgarzadeh (5719)<br>Amin Ojani (2716)

12 points are required to pass.
12-16: 3
16-20 : 4
20-24 : 5
Please start each new problem at the top of a page! Only use one side of each paper!

1) Figure 1 shows an amplifier schematic. Determine the input-referred noise voltage. Consider only the thermal noise sources and ignore the gate noise of the transistor. $\left(\lambda=\gamma_{\text {body effect }}=0\right)$

Hint: $\overline{i_{n, M 1}^{2}}=4 k T \gamma g_{m}$


Fig. 1. An amplifier schematic.
2) A common-source low noise amplifier (LNA) with feedback is shown in Fig. 2. $R_{S}$ is the input source resistance. Assume that the transistors are long-channel devices and $\lambda=0$.


Fig. 2. A CS low-nose amplifier (LNA).
(a) Determine the input impedance ( $R_{i n}$ ) of the LNA.
(b) Calculate the voltage gain of the LNA (i.e., $V_{\text {out }} / V_{\text {in }}$ ) after matching if $R_{F}=25 R_{S}$.
(c) Derive an expression for the output noise of the LNA contributed by $R_{S}$ after matching. Assume $R_{F} \gg R_{S}$.
3) A single-balanced mixer is shown in Fig. 3. Assume that the switching transistors $M_{1}$ and $\mathrm{M}_{2}$ are ideal switches with zero on-resistance and $\lambda=0$.


Fig. 3. A single-balanced mixer.
(a) Derive an expression for the conversion gain of this mixer.
(b) Derive an expression for the noise figure of this mixer. Assume the switching transistors do not generate noise. The total noise is contributed by transistor $\mathrm{M}_{3}$, load resistors $R$ and source resistor $R_{S}$ connected to the RF input (is not shown in the figure). Consider only the thermal noise sources and ignore the gate noise of the transistor.

## Hints:

i) $\overline{i_{n, M}^{2}}=4 k T \gamma g_{m}$
ii) $V_{L O}(t)=\frac{4}{\pi} \cos \omega_{L O}(t)-\frac{4}{3 \pi} \cos 3 \omega_{L O}(t)+\frac{4}{5 \pi} \cos 5 \omega_{L O}(t)-\ldots$
4) Two identical ideal integrators are connected in a feedback loop as shown in Fig. 4. The transfer function of each integrator is $H(s)=k / s$.
(a) Determine the closed-loop transfer function (i.e., $\frac{Y}{X}(s)$ ).
(b)Using Bode plot, show the relationship between the phase and the magnitude for different frequencies.
(c)Write a differential equation describing the closed-loop system and show that the system oscillates generating a sinusoidal output. Determine the oscillation frequency with respect to $k$.


Fig. 4. Two integrators in a feedback loop.
5)
(a) Explain how a type-I PLL operates as a FSK demodulator, if the VCO control voltage is considered as the output.
(b) Figure 5 show the waveforms of PFD and charge pump in a type-II PLL. Using this figure, determine the transfer function of this combination (i.e., $\frac{V_{\text {out }}}{\Delta \phi}(s)$, where $\Delta \phi$ is the phase difference between PFD inputs). Assume $I_{1}=I_{2}=I_{p}$.


Fig. 5. PFD and Charge Pump.

## TRANSISTOR EQUATIONS



PMOS


## NMOS

- Cutoff: $\quad \mathrm{I}_{\mathrm{D}}=0 \quad\left(\mathrm{~V}_{\mathrm{GS}}<\mathrm{V}_{\mathrm{TN}}\right)$
- Linear mode:

$$
I_{D}=\mu_{n} C_{o x} \frac{W}{L}\left(\left(V_{G S}-V_{T N}\right) V_{D S}-\frac{V_{D S}^{2}}{2}\right) \quad\left(\mathrm{V}_{\mathrm{GS}}>\mathrm{V}_{\mathrm{TN}}\right) \text { and }\left(\mathrm{V}_{\mathrm{DS}}<\mathrm{V}_{\mathrm{GS}}-\mathrm{V}_{\mathrm{TN}}\right)
$$

- Saturation mode:

$$
I_{D}=\frac{1}{2} \mu_{n} C_{o x} \frac{W}{L}\left(V_{G S}-V_{T N}\right)^{2}\left(1+\lambda V_{D S}\right) \quad\left(\mathrm{V}_{\mathrm{GS}}>\mathrm{V}_{\mathrm{TN}}\right) \text { and }\left(\mathrm{V}_{\mathrm{DS}}>\mathrm{V}_{\mathrm{GS}}-\mathrm{V}_{\mathrm{TN}}\right)
$$

- On-resistance in triode region: $R_{o n}=\frac{1}{\mu_{n} C_{o x} \frac{W}{L}\left(V_{G S}-V_{T N}\right)}$


## PMOS

- Cutoff: $\quad \mathrm{I}_{\mathrm{D}}=0 \quad\left(\mathrm{~V}_{\mathrm{GS}}<\left|\mathrm{V}_{\mathrm{TP}}\right|\right)$
- Linear mode:
$I_{D}=\mu_{p} C_{o x} \frac{W}{L}\left(\left(V_{S G}-\left|V_{T P}\right|\right) V_{S D}-\frac{V_{S D}^{2}}{2}\right) \quad\left(\mathrm{V}_{\mathrm{GS}}>\left|\mathrm{V}_{\mathrm{TP}}\right|\right)$ and $\left(\mathrm{V}_{\mathrm{SD}}<\mathrm{V}_{\mathrm{SG}}-\left|\mathrm{V}_{\mathrm{TP}}\right|\right)$
- Saturation mode:

$$
I_{D}=\frac{1}{2} \mu_{p} C_{o x} \frac{W}{L}\left(V_{S G}-\left|V_{T P}\right|\right)^{2}\left(1+\lambda V_{S D}\right) \quad\left(\mathrm{V}_{\mathrm{GS}}>\left|\mathrm{V}_{\mathrm{TP}}\right|\right) \text { and }\left(\mathrm{V}_{\mathrm{SD}}>\mathrm{V}_{\mathrm{SG}}-\left|\mathrm{V}_{\mathrm{TP}}\right|\right)
$$

- On-resistance in triode region: $R_{o n}=\frac{1}{\mu_{p} C_{o x} \frac{W}{L}\left(V_{S G}-\left|V_{T P}\right|\right)}$

