

ANSWERS

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TSEK03

RADIO FREQUENCY INTEGRATED CIRCUITS

Date: 2012-05-24
Time: 14-18
Location: Kåra
Aids: Calculator, Dictionary
Teachers: Behzad Mesgarzadeh (5719)
Amin Ojani (2815)

12 points are required to pass.

12-16 : 3

16-20 : 4

20-24 : 5

**Please start each new problem at the top of a page!
Only use one side of each paper!**

1)

The gain of the first stage is $A_1 = -g_{m1}R_1$ and the gain of the second stage is $A_2 = -g_{m2}R_2$. There are two noise sources contributed by the transistors and two noise sources contributed by R_1 and R_2 . Thus the total noise at the output is:

$$N_o = 4kTR_S\Delta f \cdot A_1^2A_2^2 + 4kT\gamma g_{m1}\Delta f \cdot R_1^2 \cdot A_2^2 + 4kT\gamma g_{m2}\Delta f \cdot R_2^2 + 4kTR_1\Delta f \cdot A_2^2 + 4kTR_2\Delta f$$

The total noise at the output due to the source is: $4kTR_S\Delta f \times A_1^2A_2^2$

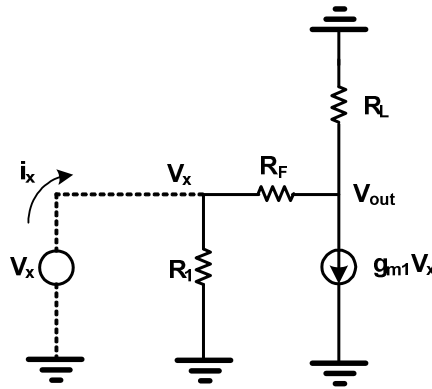
Based on these expressions the noise figure is:

$$NF = \frac{N_o}{4kTR_S\Delta f \times A_1^2A_2^2} \Rightarrow$$

$$NF = 1 + \frac{\gamma}{g_{m1}R_S} + \frac{\gamma}{g_{m1}g_{m2}R_S R_1^2} + \frac{1}{g_{m1}^2 R_S R_1} + \frac{1}{g_{m1}^2 g_{m2}^2 R_S R_1^2 R_2}$$

2)

(a) We draw the SSM and then apply a test voltage (V_x) to determine the input impedance.



The following KCL equations can be written:

$$i_x - (V_x - V_{out})/R_F - V_x/R_1 = 0 \quad (1)$$

$$g_{m1}V_x + (V_{out} - V_x)/R_F + V_{out}/R_L = 0 \quad (2)$$

From (2):

$$V_{out} = \frac{V_x(1/R_F - g_{m1})}{1/R_F + 1/R_L} \quad (3)$$

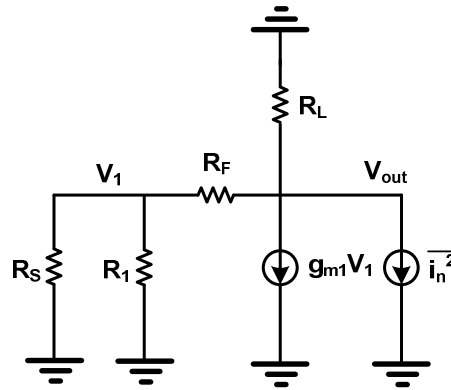
Replacing (3) into (1) results in:

$$i_x - \left(-\frac{1/R_F - g_{m1}}{1 + R_F/R_L} + \frac{1}{R_1} + \frac{1}{R_F} \right) V_x = 0$$

Thus:

$$R_{in} = \frac{V_x}{i_x} = \frac{1}{\frac{g_{m1} - 1/R_F}{1 + R_F/R_L} + \frac{1}{R_1} + \frac{1}{R_F}}$$

(b) Considering the drain noise of M1, we can draw the SSM as following:



Let's denote $R = (R_1 \parallel R_2)$. We write:

$$V_1 = V_{out} \frac{R}{R_F + R} \quad (1)$$

and

$$V_{out} = -(R_L \parallel (R_F + R))(i_n + g_{m1} V_1) \quad (2)$$

Replacing (1) into (2):

$$\overline{V_{out}^2} = \frac{(R_L \parallel (R_F + R))^2}{\left[1 + \frac{g_{m1} R}{R_F + R} \cdot (R_L \parallel (R_F + R)) \right]^2} \cdot \overline{i_n^2}$$

After some simplifications, replacing the expression for $\overline{i_n^2}$ gives:

$$\overline{V_{out}^2} = \frac{4kT\gamma g_m R_L^2 (R_F + R)^2 \Delta f}{(R_F + R + R_L + g_{m1} R R_L)^2}$$

3) Proof has been provided in the lecture notes.

4)

(a)

$$V_{out} = \frac{I_{in}}{Y_{tank}} \Rightarrow \frac{V_{out}}{I_{in}} = \frac{1}{Y_{tank}} = \frac{1}{1/jL\omega + j\omega C + 1/R} = \frac{jRL\omega}{j\omega L + R(1 - LC\omega^2)}$$

To determine the oscillation frequency, we should put the imaginary part of the transfer function equal to zero resulting in $\omega_{osc} = 1/\sqrt{LC}$.

(b)

$$Q = \frac{\omega_0}{2} \frac{d\Phi}{d\omega} \Big|_{\omega = \omega_0}$$

$$\Phi = 90 - \tan^{-1} \left(\frac{\omega L}{R(1 - LC\omega^2)} \right) \Rightarrow \frac{d\Phi}{d\omega} = \frac{RL + RL^2C\omega^2}{\omega^2 L^2 + R^2(1 - LC\omega^2)^2}$$

At oscillation frequency:

$$\omega = \omega_0 = 1/\sqrt{LC} \Rightarrow \frac{d\Phi}{d\omega} = 2RC$$

$$Q = \frac{\omega_0}{2} \frac{d\Phi}{d\omega} \Big|_{\omega = \omega_0} \Rightarrow Q = \frac{1}{2\sqrt{LC}} \cdot 2RC = R\sqrt{\frac{C}{L}}$$

5)

(a)

$$\frac{\phi_{out}(s)}{\phi_{in}(s)} = \frac{K_O K_D}{s + K_O K_D}$$

Closed-loop bandwidth:

$$\omega_h = K_O K_D$$

(b)

The unit step response is:

$$\phi_{out}(s) = (1/s) \cdot \frac{K_O K_D}{s + K_O K_D} = \frac{1}{s} - \frac{1}{s + K_O K_D} \Rightarrow \phi_{out}(t) = (1 - e^{-tK_O K_D}) \quad t > 0$$

The final value of $\phi_{out}(t)$ is 1. Putting the equation above to 0.95 gives:

$$t = -\frac{\ln 0.05}{K_O K_D} \approx \frac{3}{K_O K_D}$$