Tutorial 4: Oscillator Solutions

Problem 1

Suppose $H(s)$ in the negative feedback system shown below satisfies the following conditions at a frequency of ω_1: $|H(s)| = 1$, and $\angle H(s) = 170°$.

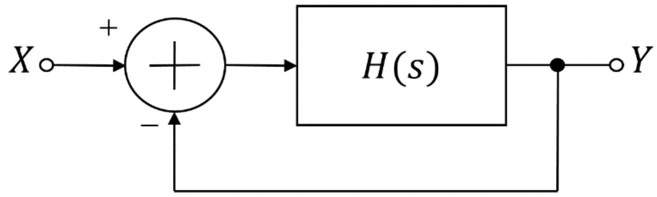


Fig. 1.1. Negative feedback system

Explain what happens to the phase and the amplitude of the output signal.

Solution:

The transfer function of the negative feedback system is expressed as

$$\frac{Y(s)}{X(s)} = \frac{H(s)}{1 + H(s)} \tag{1.1}$$

where the function $H(s)$ can be decomposed through the Euler's identity as

$$H(s) = 1 \cdot e^{j170°} = \cos 170° + j \sin 170° \tag{1.2}$$

Elaborating more for (1.1), we have

$$\frac{Y(s)}{X(s)} = \frac{H(s)[(1 + H(s)]^*}{[(1 + H(s)][(1 + H(s)]^*}$$

$$\Rightarrow \frac{Y(s)}{X(s)} = \frac{H(s) + |H(s)|^2}{1 + H(s) + H(s)^* + |H(s)|^2} \tag{1.3}$$

Substituting for $|H(s)|$ and $\angle H(s)$ in (1.3), we have

$$\frac{Y(s)}{X(s)} = \frac{(1 + \cos 170°) + j \sin 170°}{2(1 + \cos 170°)} \approx 0.5 + j5.76$$

$$\Rightarrow \frac{Y(s)}{X(s)} \approx 5.78\angle 85.04° \quad \blacksquare \tag{1.4}$$

So, if $x(t)$ is a sinusoid signal at ω_1, then the amplitude of the output signal $y(t)$ will be multiplied by 5.78 and the phase changed by 85.04 degrees approximately.

Problem 2

Prove that the series combination of the two tanks in Fig. 2.1(a) can be replaced by one tank as shown in Fig. 2.1(b).

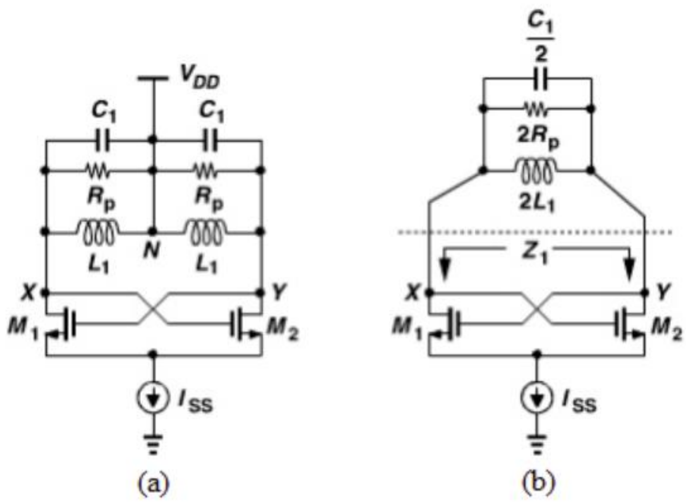


Fig. 2.1. (a) Cross-coupled oscillator. (b) with tanks merged

Solution:

The impedance observed from the nodes X and Y is equal to

$$Z_{eq} = 2\left(Z_{L1}||Z_{C1}||R_P\right) \tag{2.1}$$

Generally speaking, if we have two impedances Z_A and Z_B, then the resultant parallel impedance from Z_A and Z_B multiplied by a factor N is equal to multiply each individual impedance by the same factor N and then calculate for the resultant parallel impedance. This can be expressed as

$$N \cdot (Z_A \,||\, Z_B) \triangleq N \cdot Z_A \,||\, N \cdot Z_B \tag{2.2}$$

Demonstration:

$$N \cdot (Z_A \,||\, Z_B) = \frac{NZ_AZ_B}{Z_A + Z_B}$$

$$N \cdot (Z_A \,||\, Z_B) = \frac{N^2Z_AZ_B}{NZ_A + NZ_B} = \frac{(NZ_A)(NZ_B)}{NZ_A + NZ_B}$$

$$\Rightarrow N \cdot (Z_A \,||\, Z_B) = N \cdot Z_A \,||\, N \cdot Z_B$$

This result can be applied for more than two impedances. Then, (2.1) can also be expressed as

$$Z_{eq} = 2Z_{L1} \,||\, 2Z_{C1} \,||2R_P$$

$$\Rightarrow Z_{eq} = 2sL \,||\, {}^{2}/_{sC} \,||\, 2R_P \;\blacksquare \tag{2.3}$$

Notice that (2.3) coincide with the impedances connected in parallel in Fig. 2.1(b).

Problem 3

A negative resistance LC oscillator is shown in Fig. 3.1. The component values are L=5 nH, C = 2.5 pF, Inductor Q = 5 and $\mu_n C_{ox}$ = 110 µA/V^2.

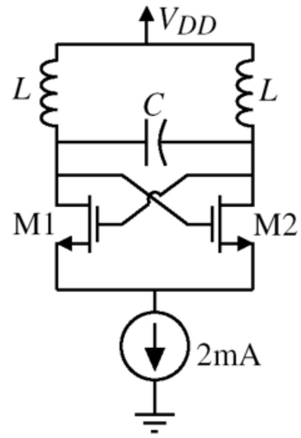


Fig. 3.1. Negative resistance LC oscillator

a) Calculate the frequency of oscillation, without and with losses in the inductor.
b) Calculate the value of negative resistance provided by the cross-coupled NMOS transistor pair to support oscillation.
c) What is the W/L ratio of $M1$ and $M2$ to achieve the required negative resistance.
d) To cover 40 MHz frequency band, the RFIC-designer would like to use a varactor, replacing the C in the schematic with a Cvar, a varactor with a tuning range from Cmin to Cmax. Calculate the tuning range of the capacitor (%). Is the required tuning range possible to achieve with a conventional CMOS varactor?

Solution:

a) The model of the resonant circuit is shown in Fig. 3.2.

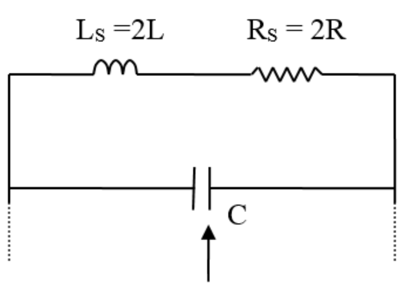


Fig. 3.2. Resonant circuit RLC

Without losses in the circuit R_s = 0. The resonance frequency in the system is equal to:

$$f_0 = \frac{1}{2\pi\sqrt{L_S C}} = \frac{1}{2\pi\sqrt{2(5nH)(2.5pF)}} \approx 1.007\ GHz \quad (3.1)$$

With losses in the circuit $R_s \neq 0$. To simplify the analysis in the circuit, we transform the RL network in series into a parallel network as shown in Fig. 3.3.

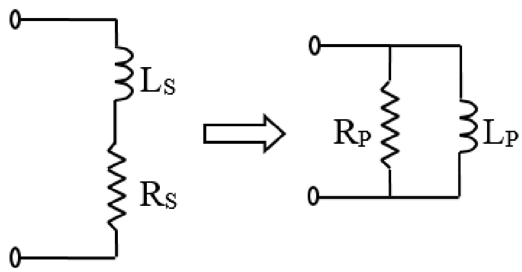


Fig. 3.3. Transformation from series to parallel RL network

In this case the, the resonant frequency can be calculated as follows

$$f_0 = \frac{1}{2\pi\sqrt{L_P C}} \tag{3.2}$$

where the inductance L_p is expressed as

$$L_p = L_s\left(1 + \frac{1}{Q^2}\right) \tag{3.3}$$

Details of the L_P derivation can be found in chapter 3 of Thomas H. Lee book: “The design of CMOS Radio Frequency Integrated Circuits”, 2nd edition. Substituting (3.3) into (3.2) and then evaluating for the resonant frequency, we have

$$f_0 = \frac{Q}{2\pi\sqrt{L_s C(Q^2+1)}} = \frac{5}{2\pi\sqrt{2(5nH)(2.5pF)(5^2+1)}} \approx\ 0.987\ GHz \tag{3.4}$$

b) The parallel resistance R_p can be expressed as

$$R_p = Q\omega_0 L_p \tag{3.5}$$

The negative resistance R_{neg} is equal to $-R_p$. So, substituting (3.3) into (3.5) and solving for $-R_p$, we have

$$R_{neg} = -(5)(2\pi \cdot 0.987\ GHz)(10\ \text{nH})\left(1 + \frac{1}{5^2}\right) \approx -322.4\,\Omega \tag{3.6}$$

c) R_{neg} seen by the RLC network can be found through the circuit shown in Fig. 3.4.

The current i_{in} can be expressed as

$$i_{in} = g_{m1}V_{gs1} \tag{3.7}$$

$$-i_{in} = g_{m2}V_{gs2} \tag{3.8}$$

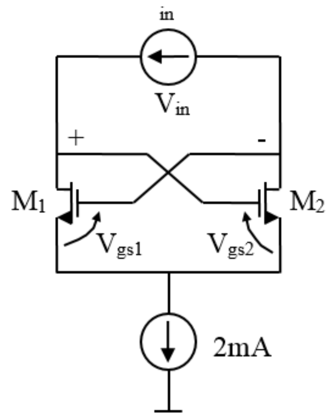


Fig. 3.4. Negative resistance seen by the RLC network

On the other hand, the voltage V_{in} can be expressed as

$$V_{in} = V_{gs2} - V_{gs1} = -i_{in}\left(\frac{1}{g_{m2}} + \frac{1}{g_{m1}}\right) \tag{3.9}$$

Assuming that g_{m1} and g_{m2} are both identical, then R_{neg} seen by the RLC network equals to

$$R_{neg} = \frac{V_{in}}{i_{in}} = -\left(\frac{1}{g_{m2}} + \frac{1}{g_{m1}}\right) = -\frac{2}{g_m} \tag{3.10}$$

Since the transconductance g_m is also expressed as $\sqrt{2\mu_n C_{ox} I_D (W/L)}$, where $\mu_n C_{ox}$ = 110 µA/V^2 and I_D = 1 mA, $-R_p$ is

$$-R_p = -\frac{2}{\sqrt{2\mu_n C_{ox} I_D (W/L)}} \tag{3.11}$$

Solving for the ratio W/L, we have

$$\frac{W}{L} = \frac{2}{\mu_n C_{ox} I_D R_p^2} \approx 175 \tag{3.12}$$

d) The tuning range of the capacitor, T = C_{max} - C_{min}, can be expressed in terms of the frequency band through the equation 8.53 in Razavi's book as

$$\Delta\omega_{osc} \approx \frac{1}{\sqrt{L_1 C_1}} \frac{C_{var2} - C_{var1}}{2C_1} = \frac{1}{\sqrt{L_1 C_1}} \frac{T}{2C_1} \tag{3.13}$$

Therefore, solving for T in (3.13), we have

$$T = \Delta\omega_{osc}\sqrt{L_1 C_1}\, 2C_1$$

$$\Rightarrow T = 2\pi(40\text{ MHz})\sqrt{(10\text{ nH})(2.5\text{ pF})}(5.0\text{ pF}) \approx 0.2\text{ pF} \tag{3.14}$$

Problem 4

A single-transistor inductor-feedback oscillator is shown in Fig. 4.1.

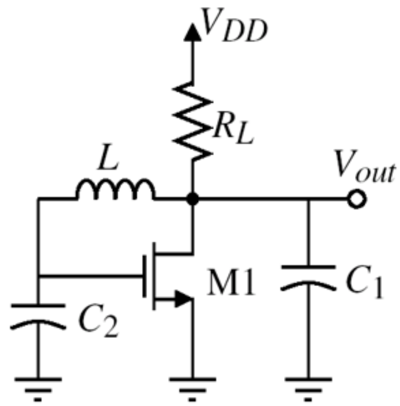
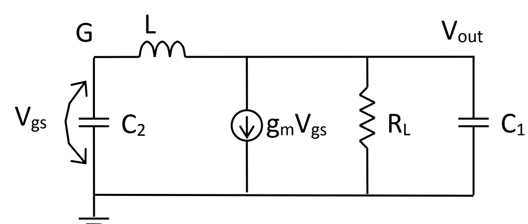


Fig. 4.1. Single-transistor inductor-feedback oscillator

Find an expression for the frequency of oscillation and the value of $g_m R_L$ necessary for oscillation. Assume that output resistance of the transistor is negligible.

Solution:

Write the node equations at the input and output.



Small-Signal Model

$$g_m V_{gs} + \frac{V_{out}}{R_L} + sC_1 V_{out} + \frac{V_{out} - V_{gs}}{sL} = 0$$

and

$$\frac{V_{out} - V_{gs}}{sL} = sC_2 V_{gs} \quad \Rightarrow \quad V_{out} = V_{gs}\left(1 + s^2 LC_2\right)$$

$$g_m V_{gs} + \left(\frac{1}{R_L} + sC_1 + \frac{1}{sL}\right)V_{out} - \frac{V_{gs}}{sL} = 0$$

$$g_m V_{gs} + \left(\frac{1}{R_L} + sC_1 + \frac{1}{sL}\right)\left(1 + s^2 LC_2\right)V_{gs} - \frac{V_{gs}}{sL} = 0$$

$$V_{gs}\left[g_m + \left(\frac{1}{R_L} + sC_1 + \frac{1}{sL}\right)\left(1 + s^2 LC_2\right) - \frac{1}{sL}\right] = 0$$

Assuming that $V_{gs} \neq 0$ we find the oscillation condition

$$g_m + \left(\frac{1}{R_L} + sC_1 + \frac{1}{sL}\right)\left(1 + s^2 LC_2\right) - \frac{1}{sL} = 0$$

$$\left(g_m + \frac{1}{R_L} - \frac{\omega^2 L C_2}{R_L}\right) + j\omega\left(C_1 - \omega^2 L C_1 C_2 + C_2\right) = 0$$

$\Downarrow$ $\Downarrow$

$$g_m + \frac{1}{R_L} = \frac{LC_2}{R_L}\omega_{osc}^2 = \frac{1}{R_L}\left(1 + \frac{C_2}{C_1}\right)$$

$$C_1 - \omega^2 L C_1 C_2 + C_2 = 0$$

$$g_m R_L = \frac{C_2}{C_1}$$

$$\omega_{osc} = \sqrt{\frac{C_1 + C_2}{L C_1 C_2}} = \frac{1}{\sqrt{LC_{eq}}}$$