Tutorial 1: Noise Solutions

Problem 1

In the amplifier schematic shown in Fig. 1.1, determine the input-referred noise voltage. Consider only the thermal noise sources and ignore the gate noise of the transistors. Neglect channel-length modulation and body effect.

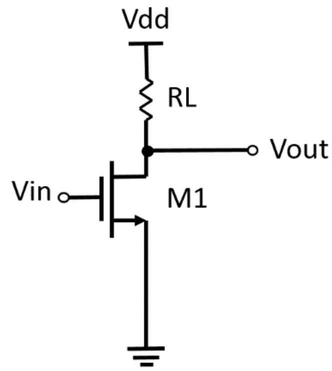

Fig. 1.1 Common-source amplifier

Solution:

The input-referred noise is calculated as follows

$$\overline{V_{n,in}^2} = \overline{V_{n,out}^2} / {A_0}^2 , \tag{1.1}$$

where A_0 is the voltage gain and $\overline{V_{n,out}^2}$ the total noise at the output of the circuit, respectively. Including the thermal noise sources in the circuit, we have:

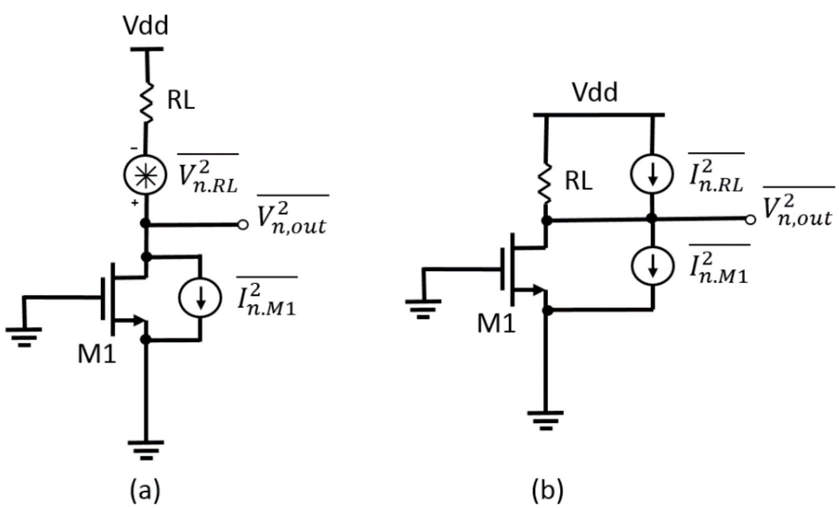

Fig. 1.2 Noise circuit representation (a) RL voltage source (b) RL current source

From Fig. 1.2, the noise contribution from M1 and RL is calculated as follows

$$\overline{V_{n,out,M1}^2} = 4kT\gamma g_m RL^2 \quad (1.2)$$

$$\overline{V_{n,out,RL}^2} = 4kTRL \quad (1.3)$$

where γ "gamma" is the excess noise contribution coefficient, equal to 2/3 for long- and 2 for short-channel transistors, g_m the transconductance, and T the absolute temperature in Kelvins. Hence, the total output PSD noise contribution is equal to:

$$\overline{V_{n,out}^2} = \overline{V_{n,out,M1}^2} + \overline{V_{n,out,RL}^2}$$

$$\overline{\Rightarrow V_{n,out}^2} = 4kTRL \cdot (\gamma g_m RL + 1) \quad (1.4)$$

Calculating now for A_0 through the small-signal model shown in Fig. 1.3.

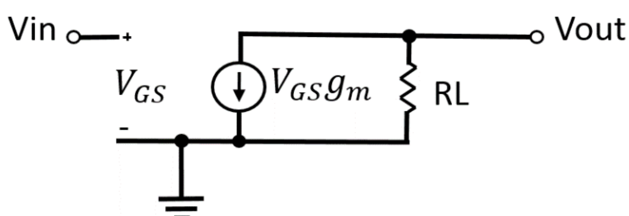

Fig. 1.3 Small-signal model

From Fig. 1.3, we have

$$\Rightarrow \; {A_0}^2 = g_m^2 R_L^2 \quad (1.5)$$

Finally, substituting (1.4) and (1.5) into (1.1), the input-referred noise is equal to:

$$\overline{V_{n,in}^2} = \frac{4kTRL \cdot (\gamma g_m RL + 1)}{g_m^2 R_L^2} = \frac{4kT \cdot (\gamma g_m RL + 1)}{g_m^2 RL} \; \blacksquare$$

Problem 2

Determine the noise figure of the stages below with respect to a source impedance of R_S. Neglect body effect, but not channel-length modulation. Assume the current sources I_1, I_2 are noiseless.

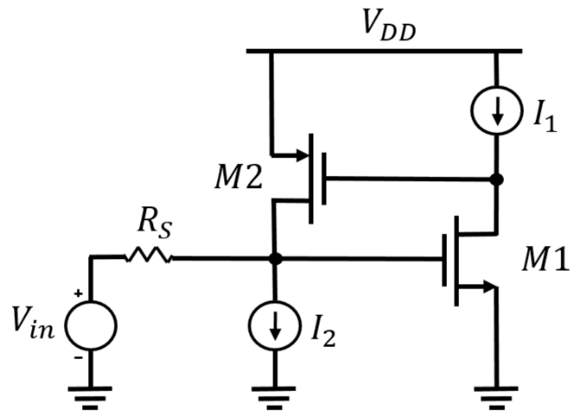

Fig. 2.1 Stages for NF calculation

Solution:

Identifying the noise sources in the circuit.

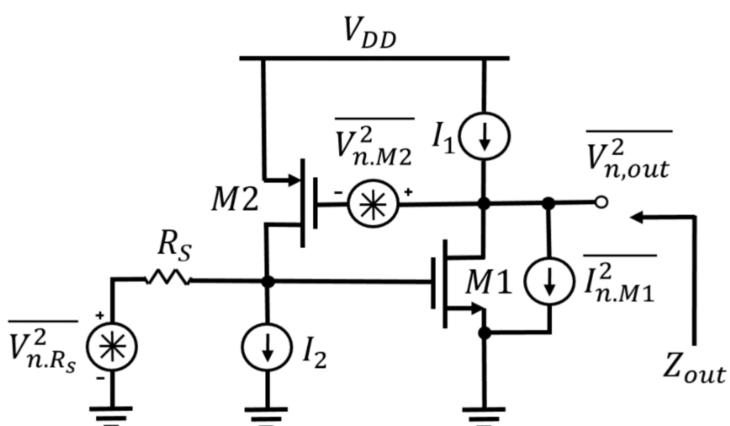

Fig. 2.2 Noise circuit representation

The noise figure is defined as

$$NF = 1 + \frac{1}{4kTR_s}\frac{\overline{V_n^2}}{{A_0}^2} \tag{2.1}$$

The total noise at the output corresponds to

$$\overline{V_n^2} = \overline{V_{n,M2}^2} + \overline{I_{n,M1}^2}{Z_{out}}^2 \tag{2.2}$$

Now, finding the gain in the circuit through the small-signal model.

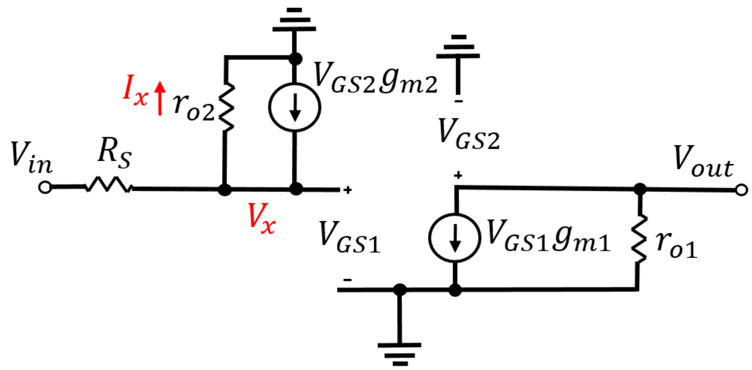

Fig. 2.3 Small-signal model

From the small-signal model in Fig. 2.3, we have

$$V_{out} = -g_{m1}V_{GS1}r_{o1} = -g_{m1}V_x r_{o1}$$

$$\Rightarrow V_x = -\frac{V_{out}}{g_{m1}r_{o1}} \tag{2.3}$$

Besides,

$$V_X = I_X r_{o2}$$

$$\Rightarrow I_X = \frac{V_X}{r_{o2}} \tag{2.4}$$

On the other hand,

$$\frac{V_{in} - V_x}{R_s} = I_X - g_{m2}V_{out} \tag{2.5}$$

Substituting (2.3) and (2.4) into (2.5), and solving for the ratio $A_0 = {V_{out}}/{V_{in}}$, then

$$\Rightarrow V_{in} + \frac{V_{out}}{g_{m1}r_{o1}} = -V_{out}R_s\left(\frac{1}{g_{m1}r_{o1}r_{o2}} + g_{m2}\right)$$

$$\Rightarrow V_{in}g_{m1}r_{o1} + V_{out} = -V_{out}\frac{R_s}{r_{o2}}(1 + g_{m1}g_{m2}r_{o1}r_{o2})$$

$$\Rightarrow A_0 = \frac{V_{out}}{V_{in}} = \frac{-g_{m1}r_{o1}r_{o2}}{r_{o2} + R_s(1 + g_{m1}g_{m2}r_{o1}r_{o2})} \tag{2.6}$$

By inspection it is observed that the output impedance in the circuit corresponds to r_{o2}.

$$Z_{out}{}^2 = r_{o1}{}^2 \tag{2.7}$$

Then, substituting (2.2), (2.6) and (2.7) into (2.1), we have for NF

$$NF = \frac{1}{4kTR_s} \frac{\overline{V_{n,M2}^2} + \overline{I_{n,M1}^2} {r_{o2}}^2}{\left[\frac{-g_{m1}r_{o1}r_{o2}}{r_{o2} + R_s(1 + g_{m1}g_{m2}r_{o1}r_{o2})}\right]^2} \tag{2.8}$$

From the transistors, we have

$$\overline{I_{n,M1}^2} = 4kT\gamma g_{m1} \tag{2.9}$$

$$\overline{V_{n,M2}^2} = \frac{4kT\gamma}{g_{m2}} \tag{2.10}$$

Finally, substituting (2.9) and (2.10) into (2.8), we have

$$NF = 1 + \frac{1}{4kTR_s} \frac{\frac{4kT\gamma}{g_{m2}} + 4kT\gamma g_{m1} {r_{o1}}^2}{\left[\frac{g_{m2}r_{o1}r_{o2}}{r_{o1} + R_s(1 + g_{m1}g_{m2}r_{o1}r_{o2})}\right]^2} \blacksquare \tag{2.11}$$

Problem 3

A two-stage amplifier is shown below. Determine the noise factor of this amplifier. Consider only the thermal noise sources and ignore the gate noise of the transistors. Assume that $R_1$and $R_2$are noiseless and ignore all the parasitics. Furthermore assume that $\lambda = 0$.

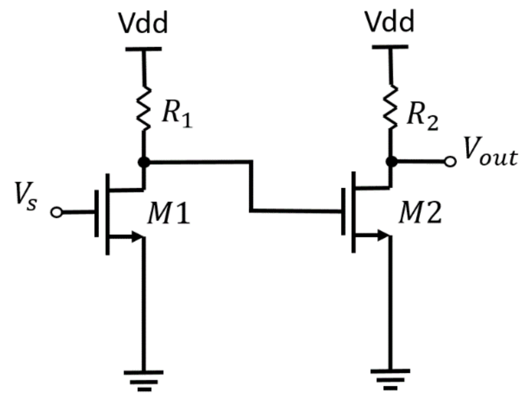

Fig. 3.1 A two-stage amplifier

Solution:

For these cascaded stages, the NF can be expressed as:

$$NF = 1 + \frac{1}{4kTR_s}\frac{\overline{V_n^2}}{{A_0}^2} \tag{2.1}$$

where $\overline{V_n^2}$ and A_0 are equal to

$$\overline{V_n^2} = 4KTg_{m1}\gamma {R_1}^2 {g_{m2}}^2 {R_2}^2 + 4KTg_{m2}\gamma {R_2}^2 \tag{2.2}$$

$$A_0 = (g_{m1}g_{m2}R_1R_2) \tag{2.3}$$

Substituting (2.2) and (2.3) into (2.1) and simplifying, we have

$$NF = 1 + \frac{4KTg_{m1}\gamma {R_1}^2 {g_{m2}}^2 {R_2}^2 + 4KTg_{m2}\gamma {R_2}^2}{4kTR_s(g_{m1}g_{m2}R_1R_2)^2}$$

$$NF = 1 + \frac{\gamma}{R_s g_{m1}} + \frac{\gamma}{R_s g_{m2}(g_{m1}R_1)^2} \blacksquare \tag{2.4}$$

Problem 4

A circuit exhibits a noise figure of 3 dB.

a) What percentage of the output noise power is due to the source resistance, R_s?
b) Repeat the problem for NF = 1 dB.

Solution:

a) The circuit can be represented as follows:

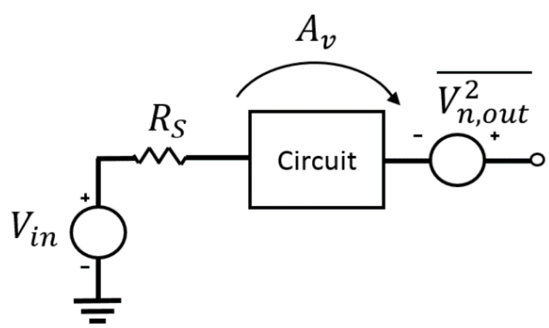

Fig. 4.1 Simplified circuit model

The NF is expressed as

$$NF = \frac{1}{4KTR_s}\frac{\overline{V_{n,out}^2}}{[|\alpha|A_v]^2} = \frac{1}{4KTR_s}\frac{\overline{V_{n,out}^2}}{{A_o}^2} \tag{4.1}$$

where α is the attenuation coefficient, and A_o is the total gain from V_{in} to V_{out}. Then, solving for $\overline{V_{n,out}^2}$ in (4.1), we have

$$\overline{V_{n,out}^2} = 4KTR_s{A_o}^2 NF \tag{4.2}$$

Dividing by the power gain, the total input referred noise can be expressed as

$$\overline{V_{n,in}^2} = 4KTR_s NF \tag{4.3}$$

A NF equal to 3 dB corresponds in linear scale to a factor of 2. Thus, the ratio in noise power with respect to the source resistance is

$$\frac{\overline{V_{n,RS}^2}}{\overline{V_{n,in}^2}} \times 100\% = \frac{4KTR_s}{(2)\cdot(4KTR_s)} \times 100\% = 50\ \% \tag{4.4}$$

b) A NF equal to 1 dB corresponds in linear scale to a factor of approximately 1.26. Thus, the ratio in noise power with respect to the source resistance is

$$\frac{\overline{V_{n,RS}^2}}{\overline{V_{n,in}^2}} \times 100\% = \frac{4KTR_s}{(1.26)\cdot(4KTR_s)} \times 100\% \approx 79.43\ \% \tag{4.5}$$