

# TSEK03: Radio Frequency Integrated Circuits (RFIC)

## Lecture 2: Noise

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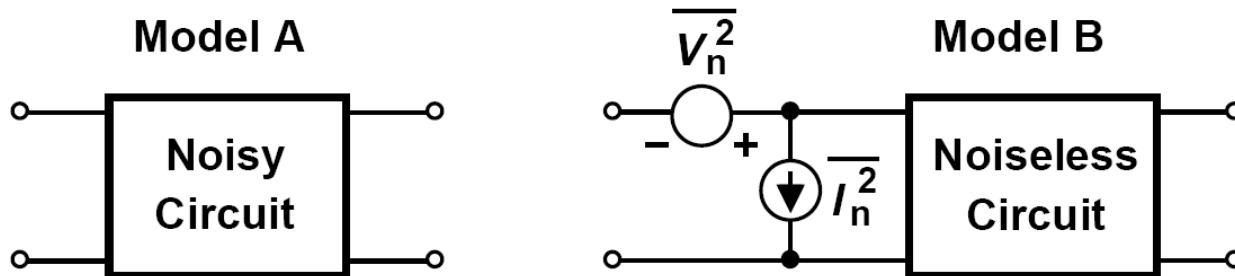
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# Overview

- Razavi: Chapter 2.3, pp. 35-58.
- Lee: Chapter 11, pp. 334-362.
- Noise: Sources of noise, noise spectrum, thermal noise,  $1/f$  (flicker) noise.
- Calculations: Noise in circuits and noise calculation, noise factor/figure, Friis' equation for cascaded noisy circuits blocks.

## 2.3.5 Representation of noise in circuits

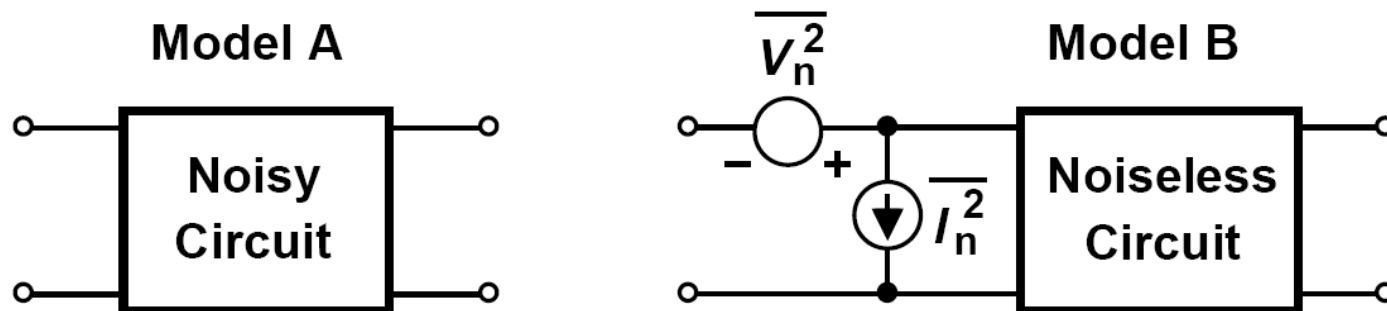
- All noise sources inside a circuit can be referred to its input, input-referred noise. The reason is that to disregard the effect of the gain on the circuit noise.
- Modeled by a series voltage source and a parallel current source:



- These sources are assumed to be correlated.

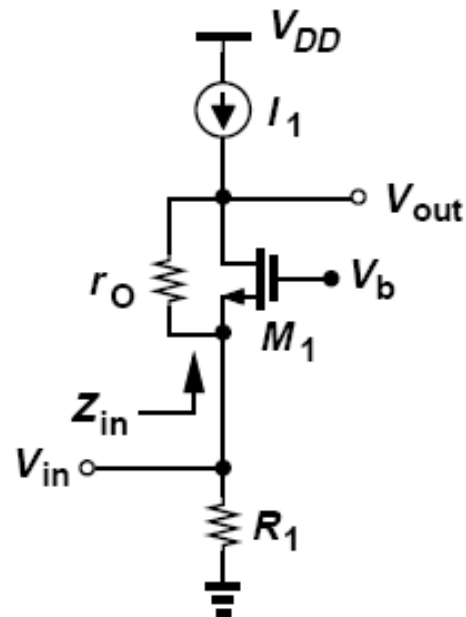
# Input-referred noise

- Voltage source: short the input port of models A and B and equate their output noise voltage (dividing the output noise by the voltage gain).
- Current source: leave the input ports open and equate the output noise voltage (dividing the output noise by the transimpedance gain).

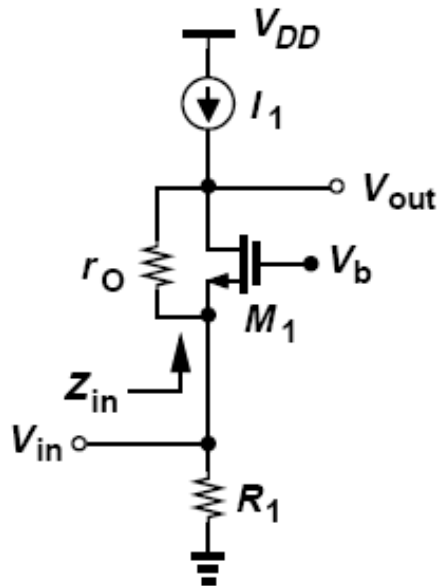


## Example 2.18

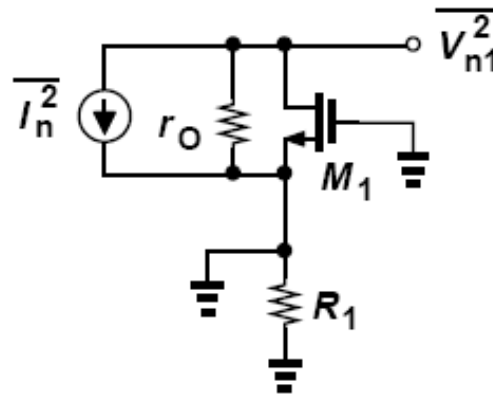
- Calculate the input-referred noise of the common-gate stage depicted in figure below (left). Assume  $I_1$  is ideal and neglect the noise of  $R_1$ .



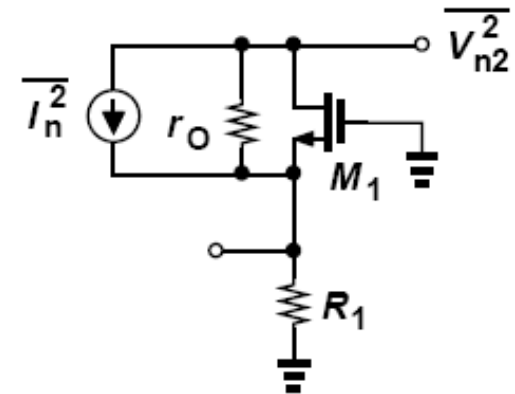
# Example 2.18 (cont)



(a) CG stage



(b) computation of input-referred noise voltage



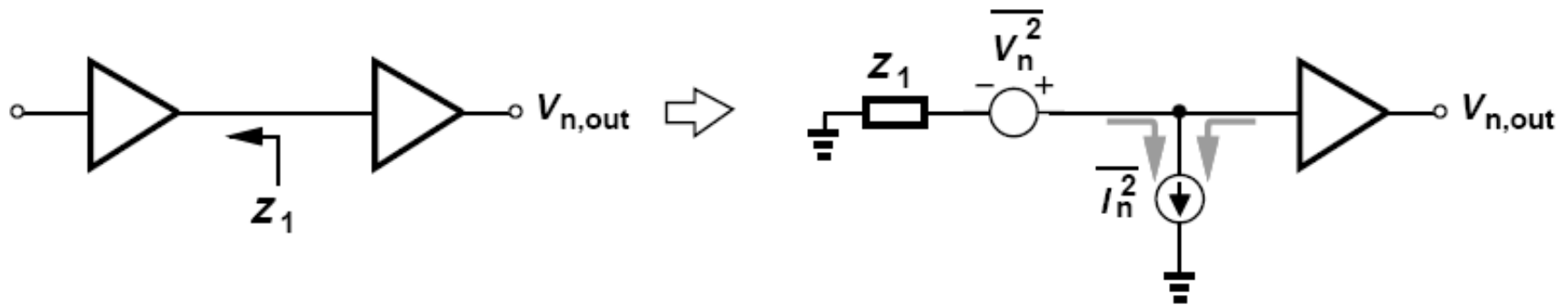
(c) computation of input-referred noise current

## Example 2.19

- Explain why the output noise of a circuit depends on the output impedance of the preceding stage.

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# Noise Figure/Factor

- Noise figure (factor) shows the noise performance of a system.

$$NF = \frac{SNR_{in}}{SNR_{out}} \quad \text{Noise Factor}$$

- It can be expressed in dB:

$$NF \Big|_{dB} = 10 \log \frac{SNR_{in}}{SNR_{out}} \quad \text{Noise Figure}$$

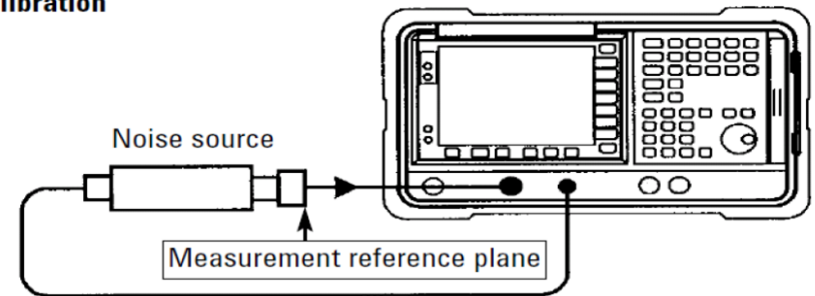
- NF depends on not only the noise of the circuit under consideration but the SNR provided by the preceding stage.
- If ideally a system adds no noise,  $F=1$ .
- If the input signal contains no noise,  $NF=\infty$ .

# NF measurements

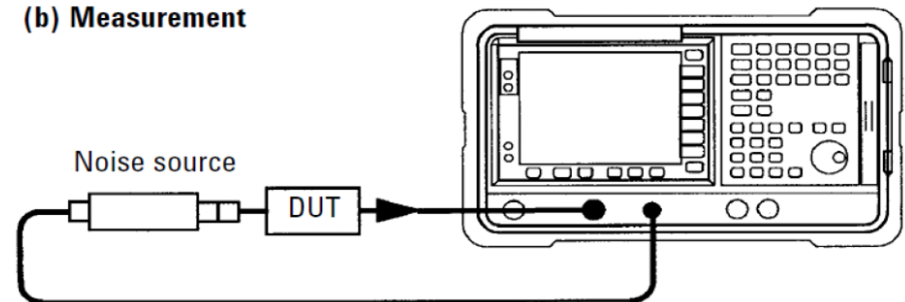


- A noise source is used to create a controlled noise level.
- Avalanche diode in reverse mode is commonly used.
- OFF: noise temp=290 K  
ON: noise temp=10 000 K

(a) Calibration



(b) Measurement

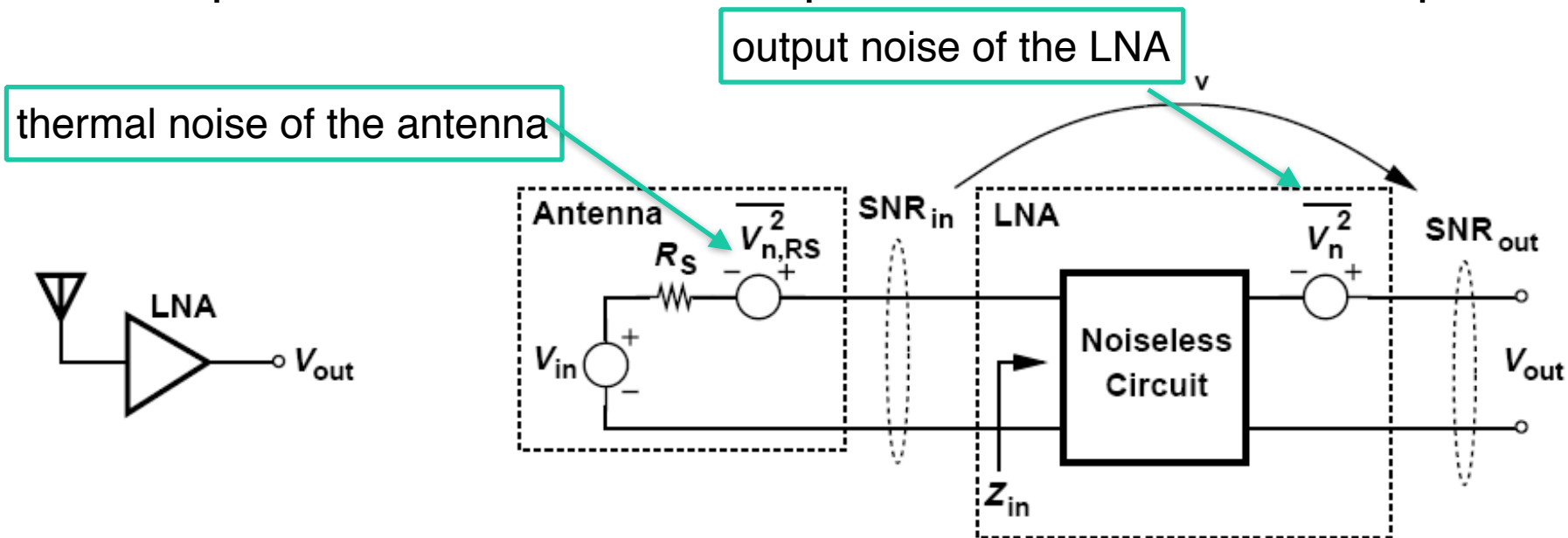


read more at:

<http://literature.cdn.keysight.com/litweb/pdf/5952-8255E.pdf>

# Calculation of Noise Figure

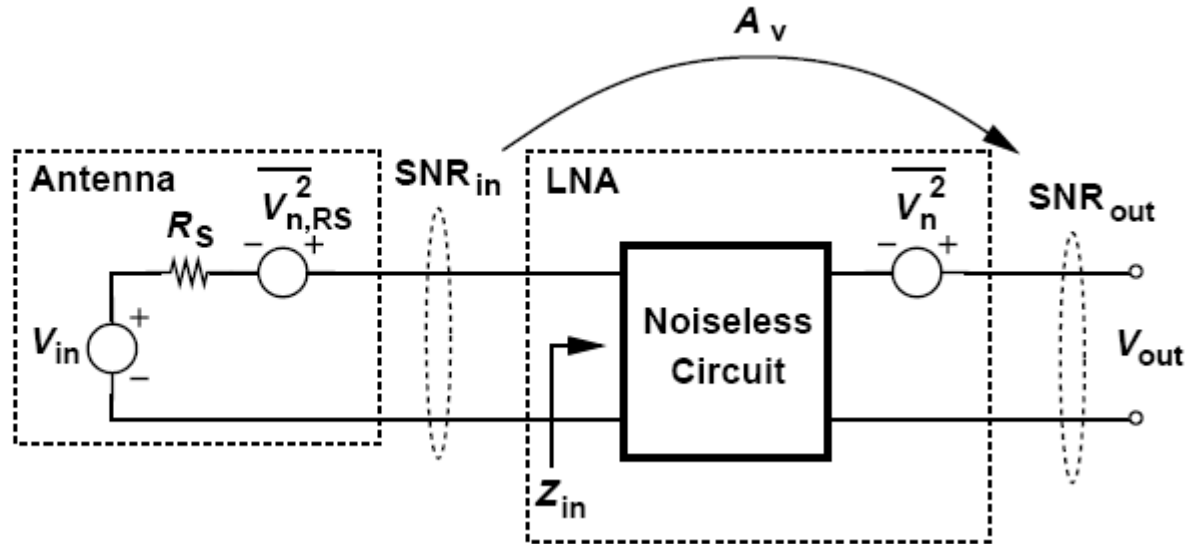
- Example: Low-noise amplifier (LNA) sensing the signal received by an antenna. NF of the LNA?
- Antenna "radiation resistance",  $R_S$ , produces noise.
- Compute  $SNR_{in}$  at the LNA input and  $SNR_{out}$  at the output.



# Noise Figure: $SNR_{in}$

- If LNA has  $Z_{in}$ ,  $V_{in}$  and  $V_{n,RS}^2$  are attenuated with  $\alpha = Z_{in}/(Z_{in} + R_S)$

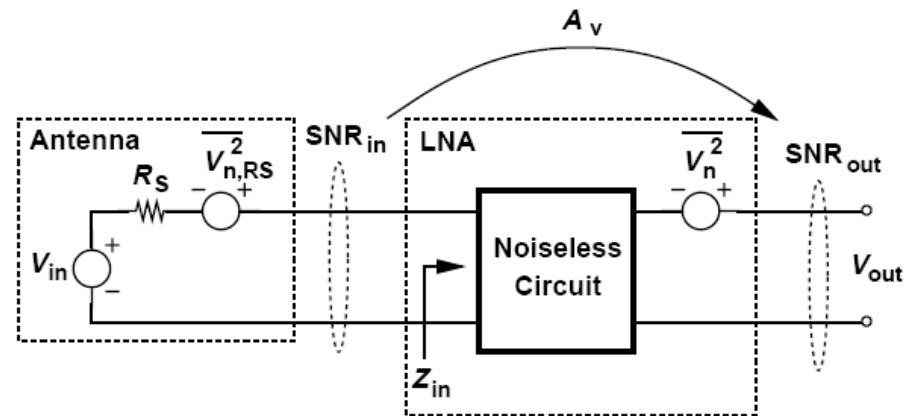
$$SNR_{in} = \frac{|\alpha|^2 V_{in}^2}{|\alpha|^2 \overline{V_{RS}^2}}$$



# Noise Figure: $SNR_{out}$

- Assume voltage gain  $A_v$  over the LNA. Output signal power is  $V_{in}^2 |\alpha|^2 A_v^2$
- Output noise (uncorrelated, can be added):
  - (a) the noise of the antenna amplified by the LNA
  - (b) the output noise of the LNA

$$SNR_{out} = \frac{V_{in}^2 |\alpha|^2 A_v^2}{V_{RS}^2 |\alpha|^2 A_v^2 + V_n^2}$$



# Noise Figure

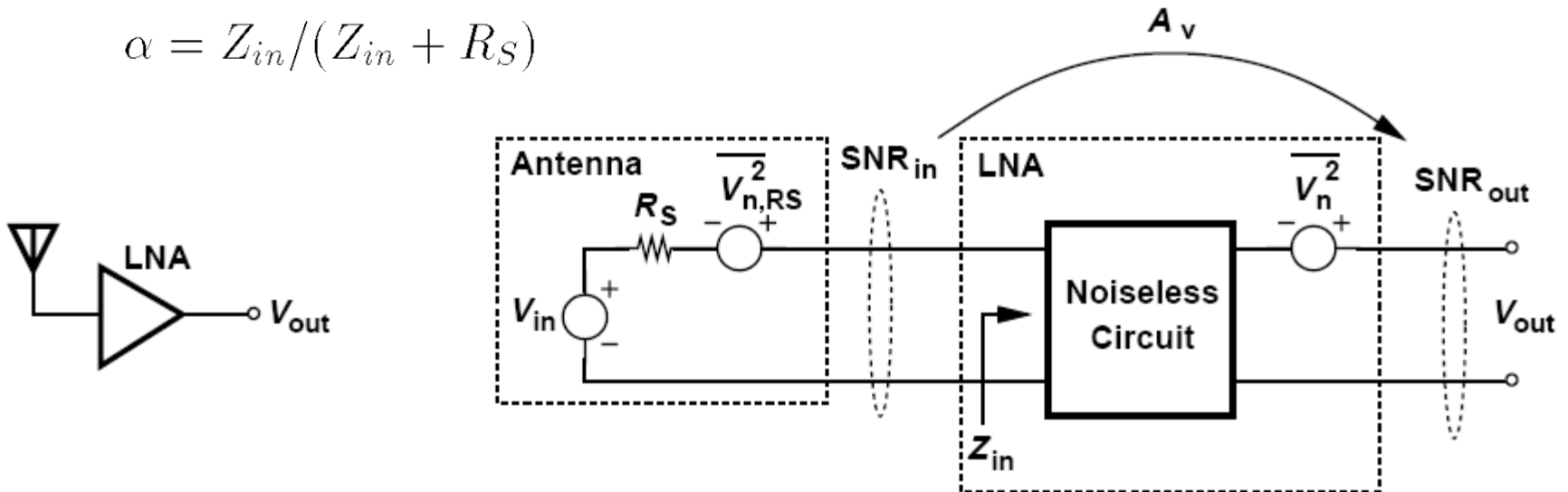
$$SNR_{in} = \frac{|\alpha|^2 V_{in}^2}{|\alpha|^2 \overline{V_{RS}^2}}$$

$$SNR_{out} = \frac{V_{in}^2 |\alpha|^2 A_v^2}{\overline{V_{RS}^2} |\alpha|^2 A_v^2 + \overline{V_n^2}}$$

$$\alpha = Z_{in} / (Z_{in} + R_S)$$

$$\begin{aligned} NF &= \frac{V_{in}^2}{4kTR_S} \cdot \frac{\overline{V_{RS}^2} |\alpha|^2 A_v^2 + \overline{V_n^2}}{V_{in}^2 |\alpha|^2 A_v^2} \\ &= \frac{1}{\overline{V_{RS}^2}} \cdot \frac{\overline{V_{RS}^2} |\alpha|^2 A_v^2 + \overline{V_n^2}}{|\alpha|^2 A_v^2} \end{aligned} \quad (2.114)$$

$$= 1 + \frac{\overline{V_n^2}}{|\alpha|^2 A_v^2} \cdot \frac{1}{\overline{V_{RS}^2}}. \quad (2.115)$$



# Noise Figure

- This results in a new definition for NF:  

$$NF = \frac{\text{(total noise at the output)}}{\text{(noise at the output due to the source impedance)}}$$

$$NF = \frac{\overline{V_{n,out}^2}}{A_0^2 \overline{V_{n,in}^2}}$$

- NF must be specified with respect to a source impedance, typically 50  $\Omega$ .
- Possible to reduce the right hand side of (2.114) to a simpler form:

$$NF = \frac{1}{4kTR_S} \cdot \frac{\overline{V_{n,out}^2}}{A_0^2} \quad (2.116)$$

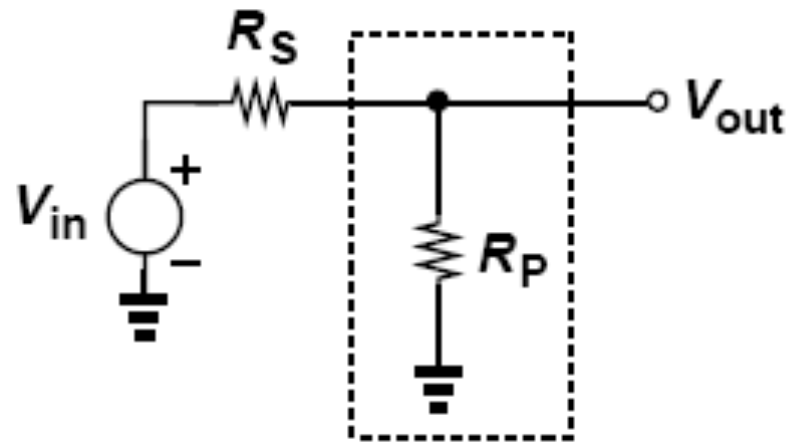
# Noise Figure Calculations

- Alt 1: Divide total output noise by the gain from  $V_{in}$  to  $V_{out}$  and normalize the result to the noise of  $R_s$ . (2.116)
- Alt 2: Calculate the output noise due to the amplifier, divide it by the gain, normalize it to  $4kTR_s$  and add 1 to the result. (2.115)
- Valid even if no actual power is transferred, since they are based on voltage quantities.  
As long as the derivations incorporate noise and signal voltages, no inconsistency arises in the presence of impedance mismatches or even infinite input impedances.



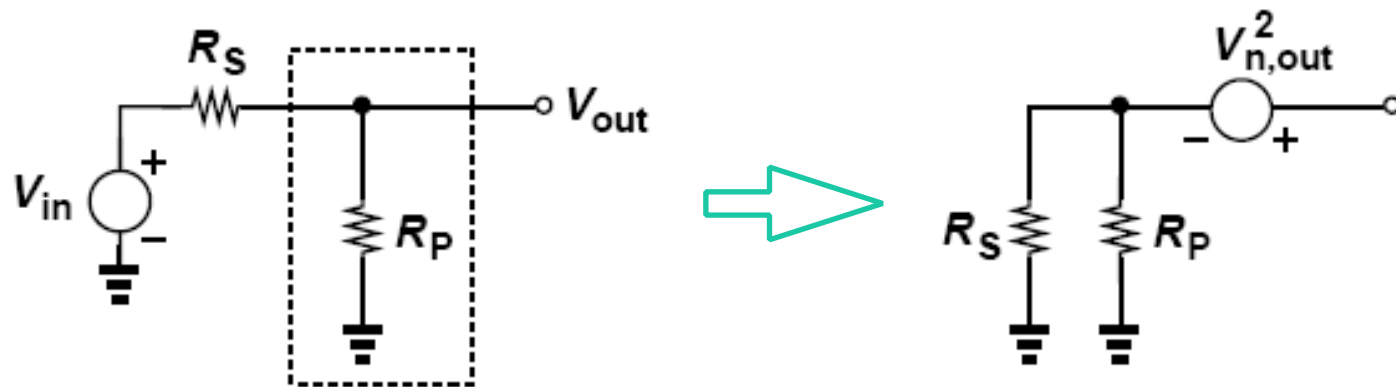
## Example 2.20

- Compute the noise figure of a shunt resistor  $R_P$  with respect to a source impedance  $R_S$ .



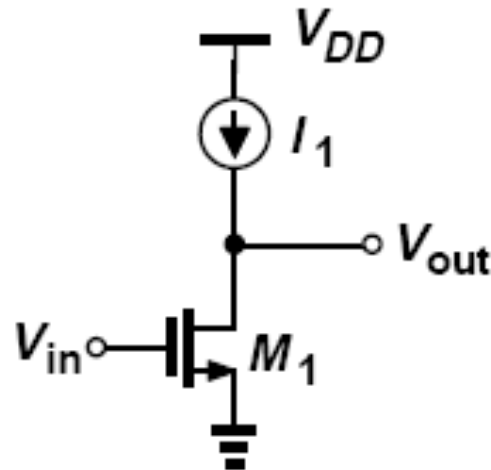
## Example 2.20 (cont)

- Compute the noise figure of a shunt resistor  $R_P$  with respect to a source impedance  $R_S$ .
- Set  $V_{in}$  to zero and add a common noise source:



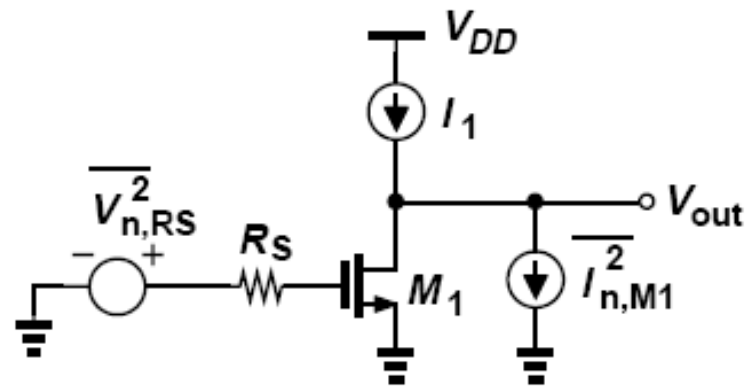
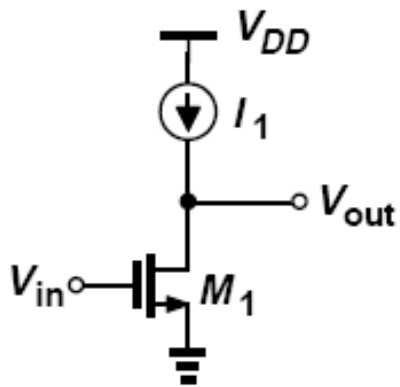
## Example 2.21

- Determine the noise figure of the common-source stage shown in below with respect to a source impedance  $R_S$ . Neglect the capacitances and flicker noise of  $M_1$  and assume  $I_1$  is ideal.



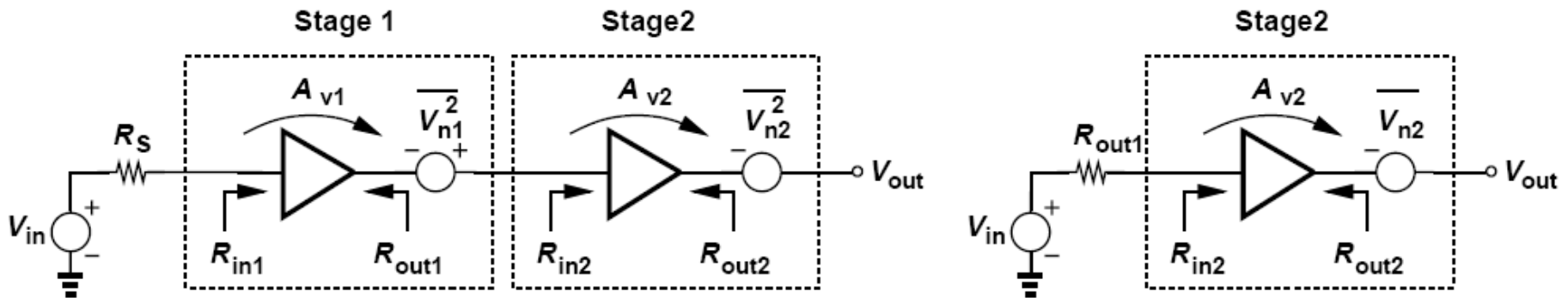
## Example 2.21 (cont)

- Determine the noise figure of the common-source stage shown in below with respect to a source impedance  $R_S$ . Neglect the capacitances and flicker noise of  $M_1$  and assume  $I_1$  is ideal.



With added noise sources

# Noise Figure of Cascaded Stages



$$NF_{tot} = 1 + (NF_1 - 1) + \frac{NF_2 - 1}{A_{P1}} + \dots + \frac{NF_m - 1}{A_{P1} \cdots A_{P(m-1)}}$$

(2.132)

Gain is power gain, which depends on the impedance of each stage.

# Noise Figure of Cascaded Stages

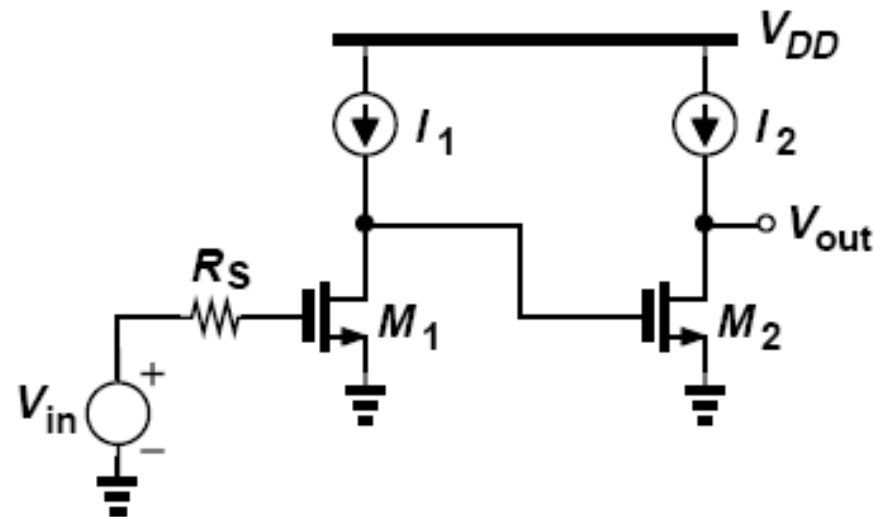
- Simplified:

$$F_{\text{sys}} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \dots + \frac{F_n - 1}{G_1 G_2 \dots G_{n-1}}$$

- Called “Friis’ equation”: the noise contributed by each stage decreases as the total gain preceding that stage increases, implying that the first few stages in a cascade are the most critical.

# Example 2.22

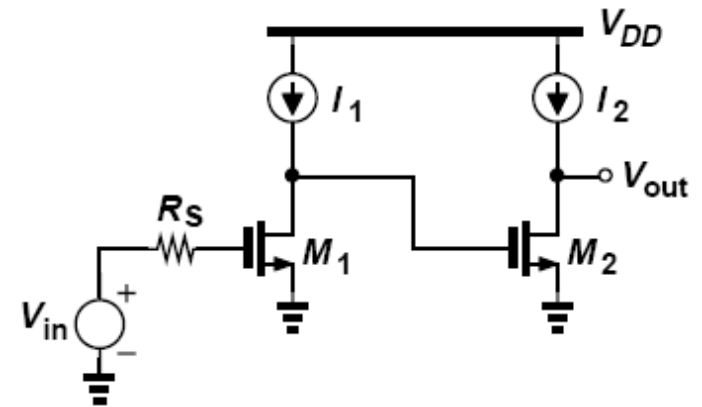
- Determine the NF of the cascade of common-source stages shown in figure below. Neglect the transistor capacitances and flicker noise.



# Example 2.22

$$R_{in1} = R_{in2} = \infty$$

$$NF = 1 + \frac{\overline{V_{n1}^2}}{A_{v1}^2} \frac{1}{4kTR_S} + \frac{\overline{V_{n2}^2}}{A_{v1}^2 A_{v2}^2} \frac{1}{4kTR_S}$$



where

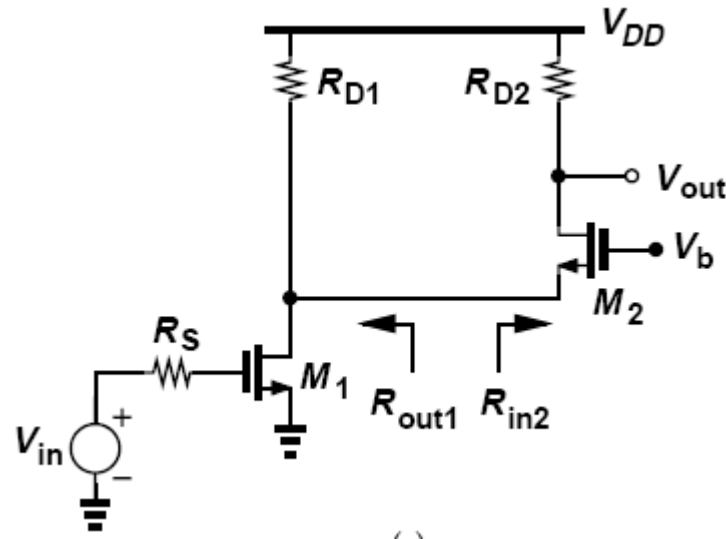
$$\overline{V_{n1}^2} = 4kT\gamma g_{m1}r_{O1}^2, \overline{V_{n2}^2} = 4kT\gamma g_{m2}r_{O2}^2, A_{v1} = g_{m1}r_{O1}, \text{ and } A_{v2} = g_{m2}r_{O2}.$$

$$NF = 1 + \frac{\gamma}{g_{m1}R_S} + \frac{\gamma}{g_{m1}^2 r_{O1}^2 g_{m2}R_S}$$

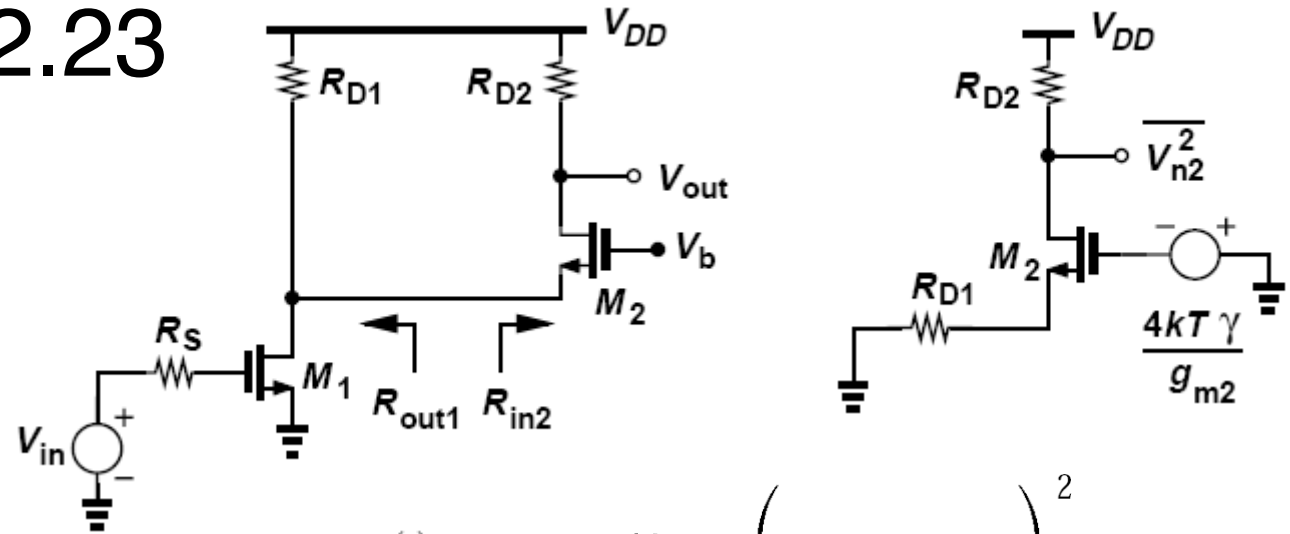


# Example 2.23

- Determine the noise figure of the circuit shown below. Neglect transistor capacitances, flicker noise, channel-length modulation, and body effect.



# Example 2.23



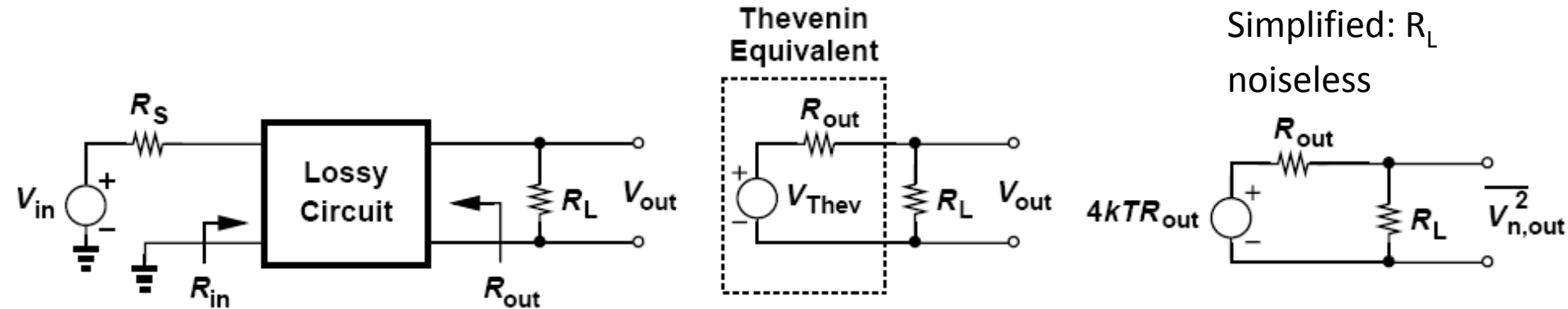
$$\overline{V_{n1}^2} = 4kT\gamma g_{m1}R_{D1}^2 + 4kTR_{D1}$$

$$\overline{V_{n2}^2} = \frac{4kT\gamma}{g_{m2}} \left( \frac{R_{D2}}{\frac{1}{g_{m2}} + R_{D1}} \right)^2 + 4kTR_{D2}$$

$$NF_{tot} = 1 + \frac{4kT\gamma g_{m1}R_{D1}^2 + 4kTR_{D1}}{g_{m1}^2 R_{D1}^2} \cdot \frac{1}{4kTR_S}$$

$$+ \frac{\frac{4kT\gamma}{g_{m2}} \left( \frac{R_{D2}}{g_{m2}^{-1} + R_{D2}} \right)^2 + 4kTR_{D2}}{g_{m1}^2 R_{D1}^2 \left( \frac{g_{m2}^{-1}}{g_{m2}^{-1} + R_{D1}} \right)^2 g_{m2}^2 R_{D2}^2} \cdot \frac{1}{4kTR_S}$$

# Noise Figure of Lossy Circuits (pp. 56-58)



- Power loss (L):

$$L = P_{in}/P_{out}$$

$$L = \frac{V_{in}^2}{V_{Thev}^2} \frac{R_{out}}{R_S}$$

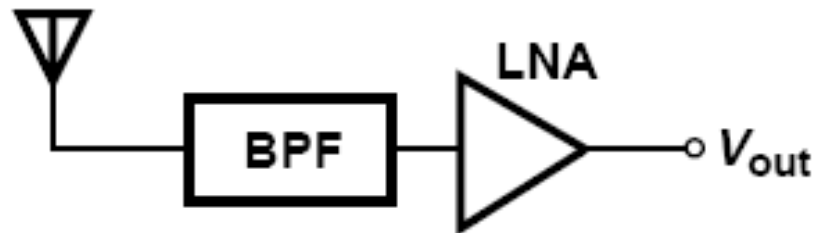
$$\overline{V_{n,out}^2} = 4kTR_{out} \frac{R_L^2}{(R_L + R_{out})^2}$$

$$A_0 = \frac{V_{Thev}}{V_{in}} \frac{R_L}{R_L + R_{out}}$$

$$\begin{aligned} NF &= 4kTR_{out} \frac{V_{in}^2}{V_{Thev}^2} \frac{1}{4kTR_S} \\ &= L. \end{aligned}$$

## Example 2.24

- The receiver shown below incorporates a front-end band-pass filter (BPF) to suppress some of the interferers that may desensitize the LNA. If the filter has a loss of  $L$  and the LNA a noise figure of  $NF_{LNA}$ , calculate the overall noise figure.



# Summary, noise

- Noise is a random process
- Noise can be represented as sources using average squared values
- Noise has spectrum - Power Spectral Density PSD - and can be filtered
- Thermal noise =  $4kTR$
- MOSFETs: channel noise, gate noise,  $1/f$  noise
- Circuit noise is usually represented by noise sources at the input: "input-referred noise"
- Noise figure and calculations
- Noise figures of cascaded stages: Friis' equation
- Noise of lossy circuits

